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Ability of goodness-of-fit tests to detect deviations from normality: the case of symmetric distributions with low values of excess kurtosis

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Abstract. The aim of this article is to verify the ability of goodness-of-fit tests (GoFTs) to detect deviations from normality. A very special case was considered: when the deviation from normality is obtained by means of symmetric distributions with low values of excess kurtosis. The first step in fulfilling the aim was to collect a set of normality-oriented tests that the source literature, especially the recent publications, recommends for use. The second step was to create a family of symmetric distributions with non-constant excess kurtosis, i.e. alternatives. Formulas for calculating the values of excess kurtosis are provided for each distribution. A relevant similarity measure was applied to compare the alternatives with the normal distribution. The third step involved carrying out a Monte Carlo simulation. The study used 20 GoFTs and 30 alternatives. The obtained results indicate that the considered GoFTs detect deviation from normality in distributions of positive excess kurtosis much better than those of negative excess kurtosis. The paper presents a set of recommended GoFTs most useful for the discussed purpose.
Keywords: normal distribution, goodness-of-fit testing, excess kurtosis modelling
JEL: C12, C13, C15

Zdolność testów zgodności do wykrywania odchyleń od normalności – przypadek rozkładów symetrycznych z małymi wartościami ekscesu

Streszczenie. Celem badania omawianego w artykule jest sprawdzenie zdolności testów zgodności do wykrywania odchyleń od normalności. Rozważano bardzo szczególny przypadek: gdy odchylenie od normalności uzyskuje się za pomocą rozkładów symetrycznych z małymi wartościami ekscesu. Badanie przeprowadzono w trzech krokach. Pierwszym było zebranie testów zgodności zorientowanych na normalność zalecanych do stosowania w literaturze przedmiotu, przede wszystkim w najnowszych publikacjach. Drugi krok polegał na utworzeniu rodziny rozkładów symetrycznych ze zmiennymi wartościami ekscesu – alternatyw. Dla każdego rozkładu podano wzory do obliczania wartości ekscesu. Aby porównać alternatywy z rozkładem normalnym, zastosowano odpowiednią miarę podobieństwa. W trzecim kroku przeprowadzono symulację Monte Carlo. W badaniu wykorzystano 20 testów zgodności i 30 opcji alternatywnych.

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Uzyskane wyniki prowadzą do wniosku, że analizowane testy zgodności znacznie lepiej wykrywają odchylenia od normalności spowodowane dodatnim ekscesem niż odchylenia wynikające z ujemnego ekscesu. Wskazano najprzydatniejsze testy zgodności.

Słowa kluczowe: rozkład normalny, testy zgodności, modelowanie ekscesu

1. Introduction

The most commonly known procedures of parametric statistical inference are applicable to parameters of normal distribution. Thus, before an analyst uses one of these procedures, he or she needs to check whether the empirical data come from the general population where normal distribution actually occurs. In other words, the analyst is required to perform a goodness-of-fit test (GoFT) for normality.

Many GoFTs are discussed in statistical literature. The most common normality test procedures available in statistical software are: the KS test (Kolmogorov, 1933; Smirnov, 1948), the LF test (Lilliefors, 1967), the CVM test (Cramer, 1928), the AD test (Anderson & Darling, 1952) and the SW test (Shapiro & Wilk, 1965). The PoweR package (Lafaye de Micheaux & Tran, 2016) from R software (R Core Team, 2021) is noteworthy, as it offers a large set of generators of pseudo-random numbers that follow probability distributions being used both frequently and sporadically. Moreover, the package provides many GoFTs for normality, uniformity and laplacity (see Section 4).

Many articles have been recently devoted to GoFTs for normality, e.g.: Afeez et al. (2018), Ahmad and Khan (2015), Aliaga et al. (2003), Arnastauskaitė et al. (2021), Bayoud (2021), Bonett and Seier (2002), Bontemps and Meddahi (2005), Brys et al. (2008), Coin (2008), Desgagné et al. (2022), Desgagné and Lafaye de Micheaux (2018), Gel et al. (2007), Gel and Gastwirth (2008), Hernandez (2021), Kellner and Celisse (2019), Khatun (2021), Marange and Qin (2019), Mbah and Paothong (2015), Mishra et al. (2019), Nosakhare and Bright (2017), Noughabi and Arghami (2011), Razali and Wah (2011), Romão et al. (2010), Sulewski (2019, 2020, 2021a), Tavakoli et al. (2019), Torabi et al. (2016), Uhm and Yi (2021), Uyanto (2022), Wijekularathna et al. (2020), Yap and Sim (2011), and Yazici and Yolacan (2007).

We focus on GoFTs for normality, which are recommended for use when alternative probability distributions are symmetric and characterised by low values of excess kurtosis (EX) $\bar{\gamma}_2 = \gamma_2 - 3$, where γ_2 is the kurtosis.

This article is the last part of a triptych devoted to detecting distributions close to normal distribution. These articles differ in their departure from normality. The first work in the triptych concerns asymmetric distributions (Sulewski, 2023b), while the second addresses distributions with undefined or constant skewness and kurtosis (Sulewski, 2024).

Symmetric alternatives can be divided into three groups. The first one comprises distributions of undefined $\bar{\gamma}_2$, the second distributions of constant $\bar{\gamma}_2$ and the third of non-constant $\bar{\gamma}_2$. The Cauchy, Voigt and Slash distributions belong to the first group. Distributions such as the arcsine, bimodal Laplace, bimodal normal, cosine, hyperbolic secant, inverted U-shaped parabolic, Laplace or double exponential, logistic or Tukey ($\lambda = 0$), raised cosine, semicircle, sine, triangular, uniform, U-shaped parabolic, and V-shaped belong to the second group. This article is devoted to alternatives that belong to the third group. The distributions of negative and positive $\bar{\gamma}_2$ values are presented in Section 3.2.

The content of Sulewski's (2023a) article was the motivation for writing this work. The article puts forward the Easily Changeable Kurtosis (ECK) distribution. The ECK enables testing the GoFT's ability to detect negative EX deviations from normality. This article shows that very popular GoFTs do not distinguish the ECK distribution of negative $\bar{\gamma}_2$ (even $\bar{\gamma}_2 = -0.3$) from the normal distribution. This is the case even when sample size $n = 30$ or 50 and significance level $\alpha = 0.05$.

In the current work, we delve into the same topic but to a much larger degree. For example, it turned out that for the ECK distribution, the power of only one of the 20 recommended GoFTs is higher than 0.06 starting from $\bar{\gamma}_2 = -0.6$, and the power of only three GoFTs starting from $\bar{\gamma}_2 = -0.7$. The results relating to the remaining 29 symmetrical alternatives are presented in Section 4 and summarised in Table 4.

As mentioned before, there are many articles devoted to testing for normality. In these articles, numerous alternatives were used, including symmetric ones. The frequently used symmetric alternatives with non-constant $\bar{\gamma}_2$ are presented below (values of the distribution parameters for which $|\bar{\gamma}_2| < 0.5$ are marked in bold):

- beta distribution for $a = 0.5,\ 0.6,\ 0.8,\ 1,\ 1.5,\ 2,\ 2.5,\ 3,\ 4,\ \mathbf{5}$;
- exponential power distribution for $a = 0.1 — 1.6,\ \mathbf{1.7 — 1.9}$;
- normal-inverse Gaussian distribution for $a = 1$;
- position-contaminated normal distribution for $a = 1,\ 3,\ 5$;
- SB distribution for $a = 0.5$;
- scale-contaminated normal distribution for $(a, 0.25)$ $(a = \mathbf{0.1},\ 0.2,\ 0.5)$,
- $(a, 2)$ $(a = 0.05,\ 0.1,\ 0.2)$, $(a,\ 4)$ $(a = 0.05,\ 0.1,\ 0.2)$;
- SU distribution for $a = 0.95,\ 1,\ 1.1,\ 1.2,\ 1.3,\ 3$;
- t Student distribution for $v = 5 — 10,\ 12,\ 14,\ 15,\ \mathbf{20},\ \mathbf{30},\ \mathbf{60},\ \mathbf{300}$;
- truncated normal distribution with $x \in [-a,\ a], a = 1,\ 2,\ \mathbf{3}$;
- Tukey distribution for $a = \mathbf{0.1},\ \mathbf{0.14},\ \mathbf{0.5},\ \mathbf{0.7},\ \mathbf{1.25},\ \mathbf{1.5},\ \mathbf{2, 5},\ \mathbf{3, 10}$.

As shown above, $\bar{\gamma}_2 \in (-0.5,\ 0.5)$ does not dominate in testing for normality. It is very interesting to see how GoFTs will respond to samples coming from alternatives close to normal distribution. In this article, we will focus on $\bar{\gamma}_2$ values close to 0. This means that we choose values of alternative parameters to obtain the desired $\bar{\gamma}_2$ values and similarity measure values of the alternative to the normal distribution. We believe that a lot of alternatives not yet used in testing for normality have been used for Monte Carlo simulations.

The aim of this article is to verify the ability of GoFTs to detect deviations from normality. A very specific case is considered: when the deviation from normality is obtained by means of symmetric distributions with low values of excess kurtosis. The first step in fulfilling the aim above is to collect a set of normality-oriented GoFTs recommended for use, mainly in the recently published source literature. The second step is to create a family of symmetric distributions with non-constant $\bar{\gamma}_2$, further called alternatives. Formulas for calculating the $\bar{\gamma}_2$ values are provided for each distribution. To compare the alternatives with the normal distribution, a relative similarity measure is applied. The third step is the Monte Carlo simulation. The study involved the use of 20 GoFTs and 30 alternatives.

2. Goodness-of-fit tests for normality

Hypothesis H_0 states: data come from normal distribution. Hypothesis H_1 negates H_0.

Table 1 presents 20 GoFTs for normality (sorted by year) recommended in scientific articles when alternatives are symmetric ($n \leq 100$). These GoFTs are used in the Monte Carlo simulations (see Section 4).

Table 1. GoFTs for normality when alternatives are symmetric ($n \leq 100$)

No.	GoFT	Recommended by
1	Anderson-Darling test (AD) (Anderson & Darling, 1952)	Wijekularathna et al. (2020), Yap and Sim (2011)
2	Shapiro-Wilk test (SW) (Shapiro & Wilk, 1965)	Desgagné et al. (2022), Mbah and Paothong (2015), Mishra et al. (2019), Nosakhare and Bright (2017), Wijekularathna et al. (2020), Yap and Sim (2011)
3	Kurtosis test (KT) (Shapiro et al., 1968)	Mishra et al. (2019)
4	D'Agostino skewness test (AS) (D'Agostino, 1970)	Mishra et al. (2019)
5	Shapiro-Francia test (SF) (Shapiro & Francia, 1972)	Nosakhare and Bright (2017)
6	D'Agostino-Pearson test (AP) (D'Agostino & Pearson, 1973)	Mishra et al. (2019), Wijekularathna et al. (2020), Yap and Sim (2011)
7	Ryan-Joiner test (RJ) (Ryan & Joiner, 1976)	Wijekularathna et al. (2020)
8	Jarque-Bera test (JB) (Jarque & Bera, 1987)	Afeez et al. (2018), Brys et al. (2008), Mbah and Paothong (2015), Yap and Sim (2011)

Table 1. GoFTs for normality when alternatives are symmetric (cont.)

No.	GoFT	Recommended by
9	1st Hosking test (H1) (Hosking, 1990)	Arnastauskaitė et al. (2021)
10	Chen-Shapiro test (CS) (Chen & Shapiro, 1995)	Desgagné et al. (2022), Romão et al. (2010)
11	Adjusted Jarque-Bera test (AJB) (Urzúa, 1996)	Wijekularathna et al. (2020)
12	Bonett–Seier test (BS) (Bonett & Seier, 2002)	Romão et al. (2010)
13	ZA Zhang-Wu test (ZA) (Zhang, 2002)	Uhm and Yi (2021)
14	ZC Zhang-Wu test (ZC) (Zhang, 2002)	Uhm and Yi (2021)
15	Gel-Miao-Gastwirth test (SJ) (Gel et al., 2007)	Romão et al. (2010), Sulewski (2019), Torabi et al. (2016), Uyanto (2022)
16	β_3^2 Coin test (β_3^2) (Coin, 2008)	Coin (2008), Romão et al. (2010)
17	Robust Jarque-Bera test (RJB) (Gel & Gastwirth, 2008)	Bayoud (2021), Sulewski (2019), Torabi et al. (2016), Uyanto (2022)
18	X_{APD} test (X_{APD}) (Desgagné & Lafaye de Micheaux, 2018)	Desgagné and Lafaye de Micheaux (2018), Wijekularathna et al. (2020)
19	Z_{EPD} test (Z_{EPD}) (Desgagné & Lafaye de Micheaux, 2018)	Desgagné and Lafaye de Micheaux (2018). Wijekularathna et al. (2020)
20	Modified Lilliefors test ($LF_{\overline{\alpha},\overline{\beta}}$) (Sulewski, 2020)	Sulewski (2020, 2021a)

Source: authors' work.

3. Similarity measure and alternatives

3.1. Similarity measure

Let $f(x;\mathbf{\theta})$ be a Probability Density Function (PDF) of the alternative with vector of parameters $\mathbf{\theta}$. Similarity measure M of the alternative to the null distribution is defined as (Sulewski, 2020)

$$M(\mathbf{\theta};\mu,\sigma) = \int_{-\infty}^{\infty} g_{\mathbf{\theta},\mu,\sigma}(x)dx, \tag{1}$$

where $g_{\mathbf{\theta},\mu,\sigma}(x) = \min\{f(x;\mathbf{\theta}),\phi(x;\mu,\sigma)\}$ for any $x \in \mathbb{R}$ and $\phi(x;\mu,\sigma)$ is the PDF of the normal distribution. The $M(\mathbf{\theta};\mu,\sigma)$ takes on the values of $[0,1]$. The $M(\mathbf{\theta};\mu,\sigma) = 1$ when the PDFs are identical. More details on distance and similarity measures can be found in Sulewski (2021a).

3.2. Alternatives

As mentioned in Section 1, the article is devoted to GoFTs for normality when the alternatives are symmetric and characterised by non-constant $\bar{\gamma}_2$. These distributions are: Bates (B), bimodal exponential power (BEP), bimodal power normal (BPN), Champernowne (CH), easily changeable kurtosis (ECK), exponential power (EP), extended normal (EN), extended Laplace (EL), extended t (ET), generalised error (GE), generalised normal (GN), Irwin-Hall (IH), plasticising component (PC), position-contaminated normal (PCN), q-Gaussian (QG), scale-contaminated normal (SCN), sinh-normal (SHN), t Student (t), two-piece skew-normal (TPSN), Tukey (TU), and U-power (UP).

There is a group of asymmetric distributions, which are symmetric for certain parameter values. The beta (BS), bimodal skew-symmetric normal (BSSN), DS normal (DSN), Edgeworth series (ES), normal-inverse Gaussian (NIG), Pearson (P), SB, SU and truncated normal (TN) distributions are analysed further in the symmetric version.

Let $\phi(x;\mu,\sigma)$ and $\Phi(x;\mu,\sigma)$ be the PDF and Cumulative Distribution Function (CDF) of $N(\mu,\sigma)$, respectively. Let $f(x;\boldsymbol{\theta})$ be the PDF of a distribution with vector of parameters $\boldsymbol{\theta}$ and $M(\boldsymbol{\theta};\mu,\sigma)$ be the similarity measure (1). K_1 denotes the modified Bessel function of the second kind and I_0 denotes the zero order Bessel function.

The Malachov inequality (Malachov, 1978) showing the relationship between skewness γ_1 and excess kurtosis $\bar{\gamma}_2$ is given by $\bar{\gamma}_2 \geq \gamma_1^2 - 2$, i.e. $\bar{\gamma}_2 \geq -2$.

Below, for each analysed distribution, we present the PDF with zero mean and standard deviation equalling 1, domain, $M(\boldsymbol{\theta};0,\sigma)$ maximum value, $\bar{\gamma}_2(\boldsymbol{\theta})$ and $\boldsymbol{\theta}(\bar{\gamma}_2)$ formulas, $\bar{\gamma}_2$ range and information about the modality. The distributions are presented in alphabetical order.

1. B distribution

$$f_B(x;a,v) = \frac{v^2}{2^{v+1}a}\sum_{k=0}^{v}\frac{(-1)^k \operatorname{sgn}\left(\frac{v}{2}-k+\frac{xv}{2a}\right)}{k!\,(v-k)!\left(v-2k+\frac{xv}{a}\right)^{1-v}},$$

$$x \in [-a,a]\ (a>0,\ v\in N_+),$$

$$M(8.077, 22; 0, 1) = 0.998,\ \bar{\gamma}_2(v) = \frac{-6}{5v},\ v(\bar{\gamma}_2) = \frac{-6}{5\bar{\gamma}_2},\ \bar{\gamma}_2 \in [-1.2,\ 0).$$

The B distribution is unimodal.

2. BEP distribution (Hassan & Hijazi, 2010)

$$f_{BEP}(x;b,p,q)=\frac{p\left|\frac{x}{b}\right|^{q}}{2b\Gamma\left(\frac{q+1}{p}\right)}\exp\left(-\left|\frac{x}{b}\right|^{p}\right),\ x\in R\ (b>0,\ p\geq 1,q\geq 0),$$

$$M\left(\sqrt{2},2,0;0,1\right)=1,\bar{\gamma}_2(p,q)=\frac{\Gamma[(q+5)/p]\Gamma[(q+1)/p]}{\Gamma[(q+3)/p]^2}-3,\ \ \bar{\gamma}_2\in[-3,3],$$

For $p=2$, we get very simple formulas:

$$\bar{\gamma}_2(q)=\frac{-2q}{q+1},\ q(\bar{\gamma}_2)=\frac{-\bar{\gamma}_2}{\bar{\gamma}_2+2}.$$

The BEP is unimodal for $q=0$ and bimodal for $q\neq 0$.

3. BPN distribution (Bolfarine et al., 2018)

$$f_{BPN}(x;a)=a\frac{2^{a-1}}{2^a-1}\phi(x;\ 0,1)\Phi(|x|;0,1)^{a-1},\ x\in R\ (a\neq 0),$$

$$M(1;0,1)=1,\ \bar{\gamma}_2(a)=\frac{\int_{-\infty}^{\infty}x^4 f_{BPN}(x;a)}{\left(\int_{-\infty}^{\infty}x^2 f_{BPN}(x;a)\right)^2}-3,\ \bar{\gamma}_2\in(-2,10.97].$$

The BPN is unimodal for $a\leq 1$ and bimodal for $a>1$.

4. BS distribution

$$f_{BS}(x;a,b)=\frac{(x+b)^{a-1}(b-x)^{a-1}}{(2b)^{2a-1}B(a,a)},\ x\in[-b,b]\ (a>0,\ b>0),$$

$$M(81.36,0.48;0,0.038)=0.999,\bar{\gamma}_2(a)=\frac{-6}{2a+3},$$

$$a(\bar{\gamma}_2)=\frac{-3\bar{\gamma}_2-6}{2\bar{\gamma}_2}\ (-2<\bar{\gamma}_2<0).$$

The BS is a unimodal distribution.

5. BSSN distribution (Hassan & El-Bassiouni, 2016)

$$f_{BSSN}(x;a,b) = \frac{2a^{1.5}(x^2+b)\exp(-ax^2)}{\sqrt{\pi}(1+2ab)},\ x \in R\ (a>0, b \geq 0),$$

$$M(22.241, 13.2018; 0, 0.15) = 0.999,$$

$$\bar{\gamma}_2(a,b) = \frac{\int_{-\infty}^{\infty} x^4 f_{BSSN}(x;a,b)dx}{\left(\int_{-\infty}^{\infty} x^2 f_{BSSN}(x;a,b)\,dx\right)^2} - 3,\ \bar{\gamma}_2 \in \left[-\frac{4}{3},\ 0\right].$$

The BSSN is unimodal e.g. for $a \geq 1,\ b \geq 1$ and bimodal e.g. for $a < 1,\ \ b < 1$.

6. CH distribution (Champernowne, 1952)

$$f_{CH}(x;a,b) = \frac{[\cosh(ax)+b]^{-1}}{\int_{-\infty}^{\infty}[\cosh(ax)+b]^{-1}dx},\ x \in R\ (a>0,\ \ b>0),$$

$$M(2.052, 3.979; 0, 1.026) = 0.992,$$

$$\bar{\gamma}_2(b) = \frac{\int_{-\infty}^{\infty} x^4 f_{CH}(x;a,b)dx}{\left(\int_{-\infty}^{\infty} x^2 f_{CH}(x;a,b)dx\right)^2} - 3,\ \ \bar{\gamma}_2 \in (-1.2, 2).$$

The CH is a unimodal distribution.

7. DSN distribution (Sulewski, 2021b)

$$f_{DSN}(x;a,b) = (3ax^2+b)\ \phi(ax^3+bx;\ 0,1),$$
$$x \in R\ (a \geq 0, b \geq 0, a+b > 0),$$

$$M(0,1;0,1) = 1,\ \bar{\gamma}_2(a,b) = \frac{\int_{-\infty}^{\infty} x^4 f_{DSN}(x;a,b)dx}{\left(\int_{-\infty}^{\infty} x^2 f_{DSN}(x;a,b)dx\right)^2} - 3,\ \ \bar{\gamma}_2 \in [-1.709, 0].$$

The DSN is unimodal e.g. for $a = 1,\ b \geq 2$ and bimodal e.g. for $a \leq 1,\ b \leq 1$.

8. ECK distribution (Sulewski, 2023a)

$$f_{ECK}(x;a,p) = \frac{\left(1-\frac{x^2}{a^2}\right)^p}{aB(0.5,p+1)},$$

$$x \in \begin{cases}[-a,a]\ (p \geq 0)\\(-a,a)(p<0)\end{cases}\ (a>0,p>-1),$$

$$M(8.326, 160.7;\ 0, 0.463) = 0.999,\ \bar{\gamma}_2(p) = \frac{-6}{2p+5},$$

$$p(\bar{\gamma}_2) = \frac{-5\bar{\gamma}_2 - 6}{2\bar{\gamma}_2}\ (-2 < \bar{\gamma}_2 < 0).$$

The ECK is a unimodal distribution.

9. EL distribution

$$f_{EL}(x;a) = \frac{1+a|x|}{2+2a}\exp(-|x|),\ x \in R\ (a \geq 0), M(2.155;\ 0, 1.968) = 0.964,$$

$$\bar{\gamma}_2(a) = \frac{3(a^2+6a+1)}{(3a+1)^2},\ a(\bar{\gamma}_2) = \frac{2\sqrt{18-6\bar{\gamma}_2} - 3\bar{\gamma}_2 + 9}{9\bar{\gamma}_2 - 3},\ \bar{\gamma}_2 \in \left[\frac{1}{3},\ 3\right].$$

The EL is unimodal for $a \in [0,1]$ and bimodal for $a > 1$.

10. EN distribution

$$f_{EN}(x;a) = \frac{1+ax^2}{1+a}\phi(x;0,1),\ x \in R\ (a \geq 0),$$

$$M(0;0,1) = 1,\ \bar{\gamma}_2(a) = -12\left(\frac{a}{3a+1}\right)^2,$$

$$a(\bar{\gamma}_2) = \frac{\sqrt{-\bar{\gamma}_2}}{2\sqrt{3} - 3\sqrt{-\bar{\gamma}_2}}\ \left(-\frac{4}{3} < \bar{\gamma}_2 < 0\right).$$

The EN is unimodal for $c \in [0,\ 0.5]$ and bimodal for $c > 0.5$.

11. EP distribution (Desgagné et al., 2022)

$$f_{EP}(x;a) = \ 0.5a^{-1/a}\Gamma^{-1}\left(1+\frac{1}{a}\right)\exp\left[-\frac{(|x|)^a}{a}\right],\ x \in R\ (a>0),$$

$$M(2;0,1)=1,\ \bar{\gamma}_2(a)=\frac{\Gamma\left(\frac{5}{a}\right)\Gamma\left(\frac{1}{a}\right)}{\Gamma\left(\frac{3}{a}\right)^2}-3,\ \ \bar{\gamma}_2 \in [-1.2,\ 0) \vee (0,\ \infty).$$

The EP is a unimodal distribution.

12. ES distribution (Aliaga et al., 2003)

$$f_{ES}(x;\ \bar{\gamma}_2)=\frac{1}{24}\phi(x;0,1)(\bar{\gamma}_2x^4-6\bar{\gamma}_2x^2+3\bar{\gamma}_2+24),\ \ x \in R\ (\bar{\gamma}_2 \geq -2),$$

$$M(0;\ 0,1)=1.$$

The ES is a unimodal distribution. The PDF formula above is introduced in the Appendix.

13. ET distribution

$$f_{ET}(x;a,v)=\frac{n+x^2(1+a+an)}{(a+1)(n+x^2)}\frac{(1+x^2v^{-1})^{-0.5(v+1)}}{\sqrt{v}B(0.5,0.5v)},$$
$$x \in R\ (a \geq 0,\ v>0),$$

$$M(0.071, 149.141;\ 0, 1.072)=1,$$
$$\bar{\gamma}_2(a,v)=\frac{36a-12a^2v+78a^2+6}{(v-4)(3a+1)^2}(v>4),\ \bar{\gamma}_2 \geq -\frac{4}{3},$$

$$v(a,\bar{\gamma}_2)=\frac{2(18a+2\bar{\gamma}_2+18a^2\bar{\gamma}_2+39a^2+12a\bar{\gamma}_2+3)}{\bar{\gamma}_2+9a^2\bar{\gamma}_2+12a^2+6a\bar{\gamma}_2},$$
$$a(v,\bar{\gamma}_2)=\frac{12\bar{\gamma}_2+2\sqrt{3(v-2)(4\bar{\gamma}_2-\bar{\gamma}_2v+6)}-3\bar{\gamma}_2v+18}{3(12\bar{\gamma}_2-4v-3\bar{\gamma}_2v+26)}.$$

The ET is unimodal for $a \in [0, 0.5]$ and bimodal for $a > 0.5$.

14. GE distribution (Giller, 2005)

$$f_{GE}(x;a) = 2^{-a-1}\Gamma^{-1}(a+1)\exp\left[-\frac{1}{2}(|x|)^{\frac{1}{a}}\right],\ x \in R\ (a > 0),$$

$$M(0.5;0,1) = 1,\ \bar{\gamma}_2(a) = \frac{\Gamma(5a)\Gamma(a)}{\Gamma(3a)^2} - 3,\ \bar{\gamma}_2 \in [-1.2, \infty).$$

The GE is a unimodal distribution.

15. GN distribution (Subbotin, 1923)

$$f_{GN}(x;a,b) = \frac{b}{2a}\Gamma^{-1}\left(\frac{1}{b}\right)\exp\left[-\left(\frac{|x|}{a}\right)^c\right],\ x \in R\ (a > 0, b > 0),$$

$$M\left(\sqrt{2}, 2; 0, 1\right) = 1,\ \bar{\gamma}_2(b) = \frac{\Gamma\left(\frac{5}{b}\right)\Gamma\left(\frac{1}{b}\right)}{\Gamma\left(\frac{3}{b}\right)^2} - 3,$$

$$\bar{\gamma}_2 \in [-1.2, 0) \vee (0, \infty).$$

The GN is a unimodal distribution.

16. IH distribution (Hall, 1927; Irwin, 1927)

$$f_{IH}(x;v) = \frac{1}{(v-1)!}\sum_{k=0}^{[x]} \frac{(-1)^k \binom{v}{k}}{(x-k)^{1-v}},\ x \in [0, v]\ (v \in N_+),$$

$$M(22; 11, 1.362) = 0.998,\ \bar{\gamma}_2(v) = \frac{-6}{5v},\ \ v(\bar{\gamma}_2) = \frac{-6}{5\bar{\gamma}_2}\ (-1.2 \le \bar{\gamma}_2 < 0).$$

The IH is a unimodal distribution.

17. NIG distribution (Barndorff-Nielsen, 1997)

$$f_{NIG}(x;a,b)=\frac{ab\exp(ab)}{\pi\sqrt{x^2+b^2}}K_1\left(a\sqrt{x^2+b^2}\right),\ x\in R\ (a>0,b>0),$$

$$M(16.815,16.85;0,1)=1,\ \ \bar{\gamma}_2(a,b)=\frac{3}{ab},$$

$$a(\bar{\gamma}_2,b)=\frac{3}{d\bar{\gamma}_2},\ b(\bar{\gamma}_2,a)=\frac{3}{a\bar{\gamma}_2}\ (\bar{\gamma}_2>0).$$

The NIG is a unimodal distribution.

18. P distribution (Pearson, 1916)

$$f_P(x;\bar{\gamma}_2)=\begin{cases}C_6(-4\bar{\gamma}_2^2x^2-8\bar{\gamma}_2^2-24\bar{\gamma}_2)^{\frac{-5\bar{\gamma}_2-6}{2\bar{\gamma}_2}} & (-2<\bar{\gamma}_2<0)\\ \phi(x;0,1) & (\bar{\gamma}_2=0)\\ C_4(\bar{\gamma}_2x^2+2\bar{\gamma}_2+6)^{\frac{-5\bar{\gamma}_2-6}{2\bar{\gamma}_2}} & (\bar{\gamma}_2>0)\end{cases},$$

where C_4, C_6 (see Appendix) are normalising constants, $M(0;0,1)=1$. The formula above is introduced in the Appendix. The P distribution is unimodal.

19. PC distribution (Sulewski, 2022)

$$f_{PC}(x;a)=a|x|^{a-1}\phi(|x|^{2a};\ 0,1),\ x\in R\ (a\geq 1),$$

$$M(1;0,1)=1,\ \bar{\gamma}_2(a)=\frac{\sqrt{\pi}\Gamma\left(0.5+\frac{2}{a}\right)}{\Gamma\left(0.5+\frac{1}{a}\right)^2}-3\ (-2<\bar{\gamma}_2<0).$$

The PC is unimodal for $a=1$ and bimodal for $a>1$.

20. PCN distribution

$$f_{PCN}(x;a)=0.5\phi(x;\ 0,1)+0.5\phi(x;a,1),x\in R\ (a\in R),$$

$$M(0;0,1) = 1,\ \bar{\gamma}_2(a) = -\left(\frac{a^2}{a^2+2}\right)^2,$$

$$a(\bar{\gamma}_2) = \sqrt{\frac{2\sqrt{-\bar{\gamma}_2} - 2\bar{\gamma}_2}{\bar{\gamma}_2 + 1}}\ (-1 \le \bar{\gamma}_2 \le 0).$$

The PCN is unimodal for $a \in [-2, 2]$ and bimodal for $a < -2$ or $a > 2$.

21. QG distribution (Tsallis, 2009)

$$f_{QG}(x;p,q) = \frac{(3-q)\sqrt{p-pq}}{2B\left(\frac{1}{2},\ \frac{1}{1-q}\right)}[1+px^2(q-1)]^{\frac{1}{1-q}},$$

$$x \in \left[\frac{\pm 1}{\sqrt{p(1-q)}}\right]\ (p > 0, 0 < q < 1),$$

$$M(0.497, 0.994; 0, 1) = 0.999,\ \ \bar{\gamma}_2(q) = \frac{6q-6}{7-5q},$$

$$q(\bar{\gamma}_2) = \frac{7\bar{\gamma}_2 + 6}{5\bar{\gamma}_2 + 6}\ \left(-\frac{6}{7} < \bar{\gamma}_2 < 0\right).$$

The QG is a unimodal distribution.

22. SB distribution (Johnson, 1949)

$$f_{SB}(x;a,b) = \frac{2ab}{b^2-x^2}\phi\left[a\ln\left(\frac{x+b}{b-x}\right);\ 0,1\right],\ x \in [-b,\ b]\ (a > 0,\ b > 0),$$

$$M(19.645, 4; 0, 0.102) = 1, \bar{\gamma}_2(a) = \frac{\int_{-\infty}^{\infty} x^4 f_{SB}(x;a,b)dx}{\left(\int_{-\infty}^{\infty} x^2 f_{SB}(x;a,b)dx\right)^2} - 3,$$

$$\bar{\gamma}_2 \in (-1.917,\ 0).$$

The SB is a unimodal distribution.

23. SCN distribution (Böhning & Ruangroj, 2002; Gleason, 1993)

$$f_{SCN}(x; a, \omega) = (1-\omega)\phi(x; 0, 1) + \frac{\omega}{a}\phi\left(\frac{x}{a}; 0, 1\right),\ x \in R\ (0 \le \omega \le 1, a > 0),$$

$$M(a, 1; 0, 1) = M(1, \omega; 0, 1) = 1,\ \ \bar{\gamma}_2(a, \omega) = \frac{3\omega(1-\omega)(a^2-1)^2}{(1-\omega+\omega a^2)^2},$$

$$\omega(\bar{\gamma}_2, a) = \frac{2\bar{\gamma}_2 + \sqrt{a^4 - 2a^2 - 4a^2\bar{\gamma}_2 + 1} - a^2 + 1}{2(1-a^2)(\bar{\gamma}_2 + 1)},$$

$$a(\bar{\gamma}_2, \omega) = \sqrt{\frac{(\omega-1)\left(\bar{\gamma}_2 + \sqrt{\frac{\bar{\gamma}_2}{\omega(1-\omega)}} + 1\right)}{\omega + \bar{\gamma}_2\omega - 1}}\ (\bar{\gamma}_2 \ge 0),$$

$$a(\bar{\gamma}_2, 0.25) = \sqrt{\frac{3 - 3\sqrt{\bar{\gamma}_2}}{\sqrt{\bar{\gamma}_2} + 3}}.$$

The SCN is a unimodal distribution.

24. SHN distribution (Rieck & Nedelman, 1991)

$$f_{SHN}(x; a, b) = \frac{2}{ab}\cosh\left(\frac{x}{a}\right)\Phi\left[\frac{2}{b}\sinh\left(\frac{x}{a}\right); 0, 1\right],\ x \in R\ (a > 0,\ b > 0),$$

$$M(0.031, 25.812; 0, 0.4) = 1,\ \bar{\gamma}_2(a) = \frac{\int_{-\infty}^{\infty} x^4 f_{SHN}(x; a)dx}{\left(\int_{-\infty}^{\infty} x^2 f_{SHN}(x; a)dx\right)^2} - 3,\ \bar{\gamma}_2 < 0.$$

The SHN is unimodal for $a \in (0, 2]$ and bathtub shape for $a > 2$.

25. Student's t distribution

$$f_T(x; v) = \frac{(1 + x^2 v^{-1})^{-0.5(v+1)}}{\sqrt{v}B(0.5, 0.5v)},\ x \in R\ (v > 0),$$

$$M(604;0,1) = 0.999,\ \bar{\gamma}_2(v) = \frac{6}{v-4}\ (v > 4),\ v(\bar{\gamma}_2) = \frac{4\bar{\gamma}_2 + 6}{\bar{\gamma}_2}\ (\bar{\gamma}_2 > 0).$$

The Student's t distribution is unimodal.

26. SU distribution (Johnson, 1949)

$$f_{SU}(x;a,b) = \frac{a}{\sqrt{2\pi}\sqrt{x^2+b^2}}\exp\left[-0.5a^2\left(a\sinh\left(\frac{x}{b}\right)\right)^2\right],$$
$$x \in R\ (a > 0,\ b > 0),$$

$$M(20.033{,}17;0,0.849) = 1,\ \ \bar{\gamma}_2(a) = \exp\left(\frac{2}{a^2}\right) + 0.5\exp\left(\frac{4}{a^2}\right) - 1.5,\ \bar{\gamma}_2 > 0.$$

The SU is a unimodal distribution.

27. TN distribution

$$f_{TN}(x;a,b) = \frac{\phi(x;0,b)}{\Phi(a;0,b) - \Phi(-a;0,b)}, x \in [-a,a]\ (a > 0,\ \ b > 0),$$

$$M(5{,}1;0,1) = 1,\ \bar{\gamma}_2(a,b) = \frac{\int_{-\infty}^{\infty} x^4 f_{TN}(x;a,b)dx}{\left(\int_{-\infty}^{\infty} x^2 f_{TN}(x;a,b)dx\right)^2} - 3,\ \bar{\gamma}_2 \in [-1.2, 0].$$

The TN is a unimodal distribution.

28. TPSN distribution (Kim, 2005)

$$f_{TPSN}(x;a,b) = \frac{2\pi\ \phi\left(\frac{x}{a};0,1\right)\ \Phi\left(\frac{b}{a}|x|;0,1\right)}{a[\pi + 2\tan^{-1}(b)]},\ x \in R\ (a > 0,\ b \geq 0),$$

$$M(1,0;0,1) = 1,\ \ \bar{\gamma}_2(a) = \frac{\int_{-\infty}^{\infty} x^4 f_{TPSN}(x;a,b)dx}{\left(\int_{-\infty}^{\infty} x^2 f_{TPSN}(x;a,b)dx\right)^2} - 3,\ \ \bar{\gamma}_2 \in [-0.388,\ 0].$$

The TPSN is unimodal for $b = 0$ and bimodal for $b > 0$.

29. TU distribution (Tukey, 1960)

$$f_{TU}(x;\lambda) = \left(Q(p,\lambda), \left(\frac{dQ(p,\lambda)}{dp}\right)^{-1}\right),$$

$$Q(p,\lambda) = \begin{cases} \dfrac{p^{\lambda} - (1-p)^{\lambda}}{\lambda} & (\lambda \neq 0) \\ \ln\left(\dfrac{p}{1-p}\right) & (\lambda = 0) \end{cases} \quad (0 \leq p \leq 1),$$

$$M\left(\frac{27}{200}; 0, \frac{29}{20}\right) = 1,\ x \in \left[-\frac{1}{\lambda}, \frac{1}{\lambda}\right] (\lambda > 0),\ \bar{\gamma}_2 \in [-1.25, 10.59],$$

$$x \in R\ \left(-\frac{1}{3} < \lambda < 0\right),\ \bar{\gamma}_2 > 0,$$

$$\bar{\gamma}_2(\lambda) = \begin{cases} 1.2\ (\lambda = 0) \\ \dfrac{\Gamma(2\lambda+1)^2[3\Gamma(2\lambda+1)^2 + \Gamma(4\lambda+1) - 4\Gamma(3\lambda+1)\Gamma(\lambda+1)]}{(2\lambda+1)^{-2}(8\lambda+2)\Gamma(4\lambda+1)[\Gamma(2\lambda+1) - \Gamma(\lambda+1)^2]^2} - 3\ (-0.25 < \lambda < 0). \end{cases}$$

The TU is a unimodal distribution.

30. UP distribution

$$f_{UP}(x;a,b) = \frac{2b+1}{2a}\left(\frac{x}{a}\right)^{2b},\ x \in [-a, a]\ (a > 0,\ b \in N^{+}),$$

$$M\left(\frac{4}{3}, 1; 0, 1\right) = 0.446,\ \bar{\gamma}_2(b) = \frac{-8b^2 - 24b - 6}{(2b+5)(2b+1)},$$

$$b(\bar{\gamma}_2) = \frac{-6\bar{\gamma}_2 + 4\sqrt{(\bar{\gamma}_2+2)(\bar{\gamma}_2+3)} - 12}{4\bar{\gamma}_2 + 8}\ (-2 \leq \bar{\gamma}_2 \leq -1.81).$$

The UP is a bathtub shape distribution.

The 30 mentioned distributions are grouped by their different properties in Table 2. It shows that most of the analysed distributions have a real domain and assume only negative $\bar{\gamma}_2$ values. The normal distribution is a special case of 21 distributions.

Table 2. Symmetric distributions with non-constant $\bar{\gamma}_2$ grouped by different properties

Property	Distribution
Finite domain	B, BS, ECK, IH, QG, SB, TN, TU, UP (9)
Infinite domain	BEP, BPN, BSSN, CH, DSN, EL, EN, EP, ES, ET, GE, GN, NIG, P, PC, PCN, SCN, SHN, SU, t, TPSN, TU (22)
$M(\mathbf{\theta}; \mu, \sigma) = 1$ for some $\mathbf{\theta}$, μ, σ	B, BEP, BPN, DSN, EN, EP, ES, ET, GE, GN, NIG, P, PC, PCN, SB, SCN, SHN, SU, t, TN, TU (21)
$\bar{\gamma}_2 < 0$	B, BS, BSSN, DSN, ECK, EN, IH, PC, PCN, QG, SB, SHN, TN, TPSN, UP (15)
$\bar{\gamma}_2 > 0$	EL, NIG, SCN, SU, t (5)
$\bar{\gamma}_2 < 0$ or $\bar{\gamma}_2 > 0$	BEP, BPN, CH, EP, ES, ET, GE, GN, P, TU (10)
Unimodal	B, BS, CH, ECK, EP, ES, GE, GN, IH, NIG, P, QG, SB, SCN, SU, t, TN, TU (18)
Bimodal	BEP, BPN, BSSN, DSN, EL, EN, ET, PC, PCN, SHN, TPSN (11)
Bathtub shape	UP (1)

Note. The number of distributions with a given property is provided in brackets.
Source: authors' work.

4. Monte Carlo simulations

For alternatives numbered from 1 to 30 (see Section 3.2), 20 large-scale experiments are performed, each dedicated to one of the GoFTs (see Section 2). Each experiment involves generating 10^4 samples of size $n = 25$. The samples come from the given alternative. Each sample is tested for normality at significance level $\alpha = 0.05$. The values of the alternative parameters are determined to obtain appropriate values of $\bar{\gamma}_2$ and similarity measure M. The power of tests (PoTs) is calculated for the given $\bar{\gamma}_2$ values.

All calculations are performed in R software using the codes presented in Table 3 and in Mathcad, in particular to calculate the values of input parameters $\mathbf{\theta}; \mu, \sigma$ for a given value of M (see formula (1)). The research tool facilitating the Monte Carlo power simulation studies for GoFTs in R, called the PoweR package (Lafaye de Micheaux & Tran, 2016), is very helpful. The 'statcompute()' function calculates the test statistic value and the p-value for the GoFT described by the 'stat.index' argument, the sample described by the 'data' argument, and the significance level described by the 'level' argument. For example, for the ZA test, the code is: statcompute(stat.index = 4, data = sample, level = 0.05). See Table 3 for more information.

Table 3. R codes of the used GoFTs

GoFT	R code	GoFT	R code
AD	ad.test	AJB	ajb.norm.test
SW	shapiro.test	BS	bonett.test
KT	kurtosis.norm.test	ZA	statcompute(stat.index = 4...)
AS	agostino.test	ZC	statcompute(stat.index = 3...)
SF	sf.test	SJ	sj.test
AP	dagoTest	β_3^2	statcompute(stat.index = 30...)
RJ	own function, see the Appendix	RJB	rjb.test
JB	jarque.test	X_{APD}	statcompute(stat.index = 36...)
H1	statcompute(stat.index = 10...)	Z_{EPD}	statcompute(stat.index = 37...)
CS	statcompute(stat.index = 26...)	$LF_{\overline{\alpha},\overline{\beta}}$	Sulewski's function, see the Appendix

Source: authors' work.

The simulation results for the alternatives are presented in alphabetical order in Tables A1–A30 in the Appendix. We assume that a GoFT detects negative or positive $\bar{\gamma}_2$ if its power is at least 0.06. These values are in bold, while the highest PoT values are underlined. Some tables present the parameters of normal distribution $\mu \neq 0, \sigma \neq 1$, which affect the similarity measure values but not the power of the tests.

Tables A1–A30 show that when alternatives are symmetric with non-constant $\bar{\gamma}_2$, the GoFTs for normality are far more efficient in detecting positive $\bar{\gamma}_2$ rather than negative $\bar{\gamma}_2$. Most tests detect an abnormal sample for $\bar{\gamma}_2 \leq -0.4$ and for $\bar{\gamma}_2 \geq 0.1$. GoFTs such as RJB, AJB, JB and SJ stand out in the case of positive $\bar{\gamma}_2$, while the Z_{EPD}, $LF_{\alpha,\beta}$ and BS tests deserve attention in the case of negative $\bar{\gamma}_2$ (especially the Z_{EPD} test). The BS and $LF_{\alpha,\beta}$ tests perform the worst in the case of positive $\bar{\gamma}_2$, while the RJB, AJB, SJ, JB and AS tests are not recommended if $\bar{\gamma}_2$ is negative.

The tests are rather effective in detecting samples coming from a bimodal distribution. For example, abnormal samples from the TPSN and BEP, tests Z_{EPD} and $LF_{\alpha,\beta}$ detect for $\bar{\gamma}_2 = -0.25, M = 0.939$ and $\bar{\gamma}_2 = -0.2, M = 0.9$, respectively, which was expected. On the other hand, the EL sample is only detected by the SF test for $EX = 0.8, M = 0.7$.

The CH samples are detected by the Z_{EPD} test for $\bar{\gamma}_2 = 0, M = 0.975$. The samples from ET and ES are detected by the Z_{EPD} test for $\bar{\gamma}_2 = -0.1, M = 0.95$ and $\bar{\gamma}_2 = -0.9, M = 0.947$, respectively. A relationship between $\bar{\gamma}_2$ and similarity measure M obviously exists, but it is not very strong. It should be noted that M is a global measure and $\bar{\gamma}_2$ is a very local measure of the curvature of the density function in the neighbourhood of the mean value.

Table 4. Summary of the results from Tables A1–A30 for the analysed alternatives

Alternative	Negative EX detection			Positive EX detection		
	$\bar{\gamma}_2$	best tests	worse tests	$\bar{\gamma}_2$	best tests	worse tests
B	−1.2	β_3^2, Z_{EPD}, CS, ZC	AJB, JB, SJ, RJB, AS	.	.	.
BEP	≤ -0.2	$LF_{\alpha,\beta}$, Z_{EPD}, CS	SJ, RJB, AJB, JB, AS	.	.	.
BPN	≤ -0.3	Z_{EPD}, $LF_{\alpha,\beta}$, BS	SJ, RJB, AJB, JB, AS	≥ 0.1	SJ, RJB, AP	BS, AD, Z_{EPD},$LF_{\alpha,\beta}$
BS	≤ -0.6	Z_{EPD}, $LF_{\alpha,\beta}$, BS	AJB, JB, RJB, AS, SJ	.	.	.
BSSN	≤ -0.4	Z_{EPD}, BS, $LF_{\alpha,\beta}$	SJ, RJB, AJB, JB, AS	.	.	.
CH	≤ 0	Z_{EPD}, $LF_{\alpha,\beta}$, BS	SJ, RJB, JB, AJB, RJ	≥ 0	AP, AJB, JB	SJ, $LF_{\alpha,\beta}$, AD
DSN	≤ -0.5	Z_{EPD}, $LF_{\alpha,\beta}$, BS	AJB, RJB, JB, SJ, AS	.	.	.
ECK	≤ -0.6	Z_{EPD}, BS, β_3^2, $LF_{\alpha,\beta}$	AJB, RJB, JB, AS, SJ	.	.	.
EL	.	.	.	≥ 0.8	AJB, SJ, RJB	BS, Z_{EPD}, $LF_{\alpha,\beta}$
EN	≤ -0.4	Z_{EPD}, BS, $LF_{\alpha,\beta}$	RJB, AJB, SJ, JB, AS	.	.	.
EP	≤ -0.4	Z_{EPD}, BS, $LF_{\alpha,\beta}$	SJ, RJB, AJB, JB, RJ, AS	≥ 0.1	SJ, RJB, AJB, JB	BS, Z_{EPD}, $LF_{\alpha,\beta}$, AD
ES	≤ -0.9	Z_{EPD}, $LF_{\alpha,\beta}$	SJ, RJB, AJB, JB, RJ, AS	≥ 0.1	SJ, RJB, AJB, JB	$LF_{\alpha,\beta}$, BS, AD, Z_{EPD}
ET	≤ -0.1	Z_{EPD}, BS, $LF_{\alpha,\beta}$	SJ, RJB, AJB, JB, RJ, AS	≥ 0.1	Z_{EPD}, ZC, BS	SJ, $LF_{\alpha,\beta}$, RJB
GE	≤ -0.4	Z_{EPD}, BS, $LF_{\alpha,\beta}$	RJB, AJB, JB, SJ, AS, RJ	≥ 0.1	SJ, RJB, AJB, JB	BS, Z_{EPD}, $LF_{\alpha,\beta}$
GN	≤ -0.4	Z_{EPD}, BS, $LF_{\alpha,\beta}$	AJB, SJ, RJB, JB, AS, RJ	≥ 0.1	SJ, RJB, AJB	BS, Z_{EPD}, AD
IH	≤ -1.2	BS, Z_{EPD}, CS, ZC	JB, RJB, AJB, SJ, AS, RJ	.	.	.
NIG	.	.	.	≥ 0.1	JB, AJB, RJB	BS, $LF_{\alpha,\beta}$, AD,
P	≤ -0.6	Z_{EPD}, β_3^2	JB, RJB, AJB, SJ, AS, RJ	≥ 0.2	JB ,AJB ,RJB	$LF_{\alpha,\beta}$, BS, AD
PC	≤ -0.3	Z_{EPD}, BS, $LF_{\alpha,\beta}$	JB, RJB, AJB, SJ, AS	.	.	.
PCN	≤ -0.4	Z_{EPD}, BS, $LF_{\alpha,\beta}$	JB, RJB, AJB, SJ, AS	.	.	.
QG	≤ -0.6	Z_{EPD}, BS, β_3^2	JB, RJB, AJB, SJ, AS	.	.	.
SB	≤ -0.5	Z_{EPD}, BS, β_3^2	JB, RJB, AJB, SJ, AS	.	.	.
SCN	.	.	.	≥ 0.2	JB, AJB, RJB	$LF_{\alpha,\beta}$, BS, AD
SHN	≤ -0.5	Z_{EPD}, BS, β_3^2	JB, RJB, AJB, SJ, AS	.	.	.
SU	.	.	.	≥ 0.2	JB, AJB, RJB, SF	$LF_{\alpha,\beta}$, AD, BS, Z_{EPD}
t	.	.	.	≥ 0.1	JB, AJB, RJB	$LF_{\alpha,\beta}$, BS, AD
TN	≤ -0.7	Z_{EPD}, BS, β_3^2	JB, RJB, AJB, SJ, AS	.	.	.
TPSN	≤ -0.25	Z_{EPD}, $LF_{\alpha,\beta}$	JB, RJB, AJB, SJ, AS	.	.	.
TU	≤ -0.6	Z_{EPD}, BS, β_3^2	JB, RJB, AJB, SJ, AS	≥ 0.1	RJB, AJB, JB	BS, $LF_{\alpha,\beta}$, AD
UP	≤ -1.81	Z_{EPD}, ZA, ZC	AS, AJB, RJB	.	.	.

Source: authors' work.

All the R codes used to produce the obtained results are available for downloading at https://github.com/PiotrSule/WS24.

5. Conclusions

The article contributes to the knowledge of GoFTs for normality. Situations when alternatives are both symmetric and of small EX values are considered. At first, GoFTs were assessed with respect to their ability to detect samples for two reasons:

- they come from general populations in which alternatives with $\bar{\gamma}_2$ values close to 0 actually occur;
- the value of the normal-alternative similarity measure is close to unity.

Having already assessed the abilities of GoFTs, 20 of them were selected from a set of GoFTs used to detect symmetric alternatives.

Next, a set of 30 alternatives was formed. These were marked out as useful in Monte Carlo studies focusing on deviation from normality. Among them were alternatives of only negative $\bar{\gamma}_2$, of only positive $\bar{\gamma}_2$, and of both negative and positive $\bar{\gamma}_2$. The alternatives in question fall into three categories with respect to their popularity in Monte Carlo studies: frequently used (e.g. BS, TN, TU), rarely used (e.g. SB, SU) and probably not used yet (e.g. B, CH). When describing a given distribution, the main emphasis was placed on defining formulas for $\bar{\gamma}_2$ and its range. In addition to the above-mentioned local measure, the values of the (global) similarity measure of the alternative to the normal distribution were determined.

The Monte Carlo study showed that the GoFTs proved much more efficient in detecting positive EX values rather than negative ones. The reason may be that the range for the negative values of $\bar{\gamma}_2$ is incomparably less extensive than the range for the positive values of $\bar{\gamma}_2$ (see Malachov inequality, mentioned in Section 3.2).

Most of the tests detected deviation from normality when $\bar{\gamma}_2 \leq -0.4$ or when $\bar{\gamma}_2 \geq 0.1$. The RJB, AJB, JB and SJ tests stand out from the rest when they deal with distributions of low positive $\bar{\gamma}_2$. In turn, when $\bar{\gamma}_2$ is small and negative, the Z_{EPD} test stands out being the best regardless of the $\bar{\gamma}_2$ value and sign. The $LF_{\alpha,\beta}$ and BS tests are close behind the Z_{EPD}. On the other hand, the $\mathrm{LF}_{\alpha,\beta}$ and BS tests cannot be recommended when dealing with alternatives of a small positive $\bar{\gamma}_2$. The RJB, AJB, SJ, JB and AS tests cannot be recommended when dealing with alternatives of a small positive $\bar{\gamma}_2$.

What is noteworthy is that RJB, AJB, JB and SJ tests recommended for detecting small positive $\bar{\gamma}_2$ cannot be recommended when dealing with small negative $\bar{\gamma}_2$ values. In contrast, the BS and LF can be recommended when dealing with small negative $\bar{\gamma}_2$ and cannot be recommended when $\bar{\gamma}_2$ is positive.

It should be mentioned here that our large-scale Monte Carlo study revealed an unexpected tendency. To our surprise, the ability to detect deviation from normality increases as $\bar{\gamma}_2$ becomes increasingly more negative. This tendency should certainly be carefully investigated in the nearest future.

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Appendix

Edgeworth series distribution

The Edgeworth series (ES) distribution is defined as

$$f_{ES}(x) = \phi(x;0,1)\left(1 + \frac{1}{3!}\chi_3 H_3(x) + \frac{1}{4!}\chi_4 H_4(x)\right), \qquad (*)$$

where $H_3(x), H_4(x)$ are the probabilist's Hermite polynomials given by

$$H_3(x) = x^3 - 3x,\ H_4(x) = x^4 - 6x^2 + 3$$

and $\chi_3 = \gamma_1,\ \chi_4 = \mu_4 - 3 = \bar{\gamma}_2$ are the cumulants, where γ_1 is the skewness. The PDF of the ES distribution, based on $(*)$, is given by

$$f_{ES}(x;\ \gamma_1,\ \bar{\gamma}_2) = \phi(x;0,1)\left(1 + \frac{1}{3!}\gamma_1(x^3 - 3x) + \frac{1}{4!}\bar{\gamma}_2(x^4 - 6x^2 + 3)\right),$$

and in the symmetric version

$$f_{ES}(x;\ \bar{\gamma}_2) = \phi(x;0,1)\left[1 + \frac{1}{24}\bar{\gamma}_2(x^4 - 6x^2 + 3)\right]\ (\bar{\gamma}_2 \geq -2).$$

Pearson distribution

The Pearson distribution is a family of continuous probability distributions. The Pearson density $f(x;\, \gamma_1,\, \bar{\gamma}_2)$ is defined as any valid solution to the following differential equation:

$$\frac{f_P'(x;\gamma_1,\bar{\gamma}_2)}{f_P(x;\gamma_1,\bar{\gamma}_2)} + \frac{x+b}{ax^2+bx+c} = 0,$$

where

$$a = \frac{2\bar{\gamma}_2 - 3\gamma_1^2}{10\bar{\gamma}_2 - 5\gamma_1^2 + 12},\;\; b = \frac{|\gamma_1|(\bar{\gamma}_2+6)}{10\bar{\gamma}_2 - 5\gamma_1^2 + 12},\;\; c = \frac{4\bar{\gamma}_2 - 3\gamma_1^2 + 12}{10\bar{\gamma}_2 - 5\gamma_1^2 + 12}.$$

The symmetric Pearson density $f(x;\bar{\gamma}_2)$ is defined as any valid solution to the following differential equation:

$$\frac{f_P'(x;\bar{\gamma}_2)}{f_P(x;\bar{\gamma}_2)} + \frac{x}{ax^2+c} = 0,$$

where

$$a = \frac{\bar{\gamma}_2}{5\bar{\gamma}_2+6},\;\; c = \frac{2\bar{\gamma}_2+6}{5\bar{\gamma}_2+6}.$$

Now the differential equation is given by

$$\frac{f_P'(x;\bar{\gamma}_2)}{f_P(x;\bar{\gamma}_2)} + \frac{x(5\bar{\gamma}_2+6)}{\bar{\gamma}_2x^2+2\bar{\gamma}_2+6} = 0,$$

so

$$f_P(x;\bar{\gamma}_2) = \exp\left(-\int \frac{x(5\bar{\gamma}_2+6)dx}{\bar{\gamma}_2x^2+2\bar{\gamma}_2+6}\right). \tag{A1}$$

Let us consider three cases determined by the sign of the discriminant (and hence the number of real roots) of the $\bar{\gamma}_2x^2+2\bar{\gamma}_2+6$.
Case 1: $\Delta = -4\bar{\gamma}_2(2\bar{\gamma}_2+6) = 0$, which means that $\bar{\gamma}_2 = 0$. From (A1), we obtain the following:

$$f_P(x) = \exp\left(-\int xdx\right) = \exp\left(-\frac{x^2}{2}+C_1\right) = C_2\exp\left(-\frac{x^2}{2}\right). \tag{A2}$$

$f(x;\bar{\gamma}_2)$ is the PDF, so $\int_{-\infty}^{\infty} f(x;\bar{\gamma}_2)dx = 1$. From the equation above, we obtain the following:

$$C_2 = \left[\int_{-\infty}^{\infty} \exp\left(-\frac{x^2}{2}\right) dx\right]^{-1} = \frac{1}{\sqrt{2\pi}},$$

thus, we obtain

$$f_P(x;\bar{\gamma}_2) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right) = \phi(x;0,1).$$

Case 2: $\Delta = -4\bar{\gamma}_2(2\bar{\gamma}_2 + 6) < 0$, which means that $\bar{\gamma}_2 > 0$. From (A1) we obtain the following:

$$f_P(x;\bar{\gamma}_2) = \exp\left(-\int \frac{x(5\bar{\gamma}_2 + 6)dx}{\bar{\gamma}_2 x^2 + 2\bar{\gamma}_2 + 6}\right) =$$

$$= \exp\left[-\frac{(5\bar{\gamma}_2 + 6)\ln(\bar{\gamma}_2 x^2 + 2\bar{\gamma}_2 + 6)}{2\bar{\gamma}_2} + C_3\right],$$

thus, we obtain

$$f_P(x;\bar{\gamma}_2) = C_4(\bar{\gamma}_2 x^2 + 2\bar{\gamma}_2 + 6)^{\frac{-5\bar{\gamma}_2 - 6}{2\bar{\gamma}_2}} \qquad (\bar{\gamma}_2 > 0).$$

where

$$C_4 = \left(\int_{-\infty}^{\infty} (\bar{\gamma}_2 x^2 + 2\bar{\gamma}_2 + 6)^{\frac{-5\bar{\gamma}_2 - 6}{2\bar{\gamma}_2}} dx\right)^{-1}. \tag{A3}$$

Case 3: $\Delta = -4\bar{\gamma}_2(2\bar{\gamma}_2 + 6) > 0$, which means that $\bar{\gamma}_2 \in (-2, 0)$. We have what follows:

$$\frac{x(5\bar{\gamma}_2 + 6)dx}{\bar{\gamma}_2 x^2 + 2\bar{\gamma}_2 + 6} = \frac{5\bar{\gamma}_2 + 6}{2\bar{\gamma}_2}\left(\int \frac{2\bar{\gamma}_2 dx}{\sqrt{\Delta} + 2\bar{\gamma}_2 x} - \int \frac{2\bar{\gamma}_2 dx}{\sqrt{\Delta} - 2\bar{\gamma}_2 x}\right)$$

$$\frac{x(5\bar{\gamma}_2 + 6)dx}{\bar{\gamma}_2 x^2 + 2\bar{\gamma}_2 + 6} = \frac{5\bar{\gamma}_2 + 6}{2\bar{\gamma}_2} \ln(\Delta - 4\bar{\gamma}_2^2 x^2).$$

Then, from (A1), we obtain

$$f_P(x;\bar{\gamma}_2) = \exp\left(-\frac{5\bar{\gamma}_2 + 6}{2\bar{\gamma}_2} \ln(-4\bar{\gamma}_2(2\bar{\gamma}_2 + 6) - 4\bar{\gamma}_2^2 x^2) + C_5\right)$$

$$f_P(x;\bar{\gamma}_2) = C_6\left(-4\bar{\gamma}_2^2x^2 - 8\bar{\gamma}_2^2 - 24\bar{\gamma}_2\right)^{\frac{-5\bar{\gamma}_2-6}{2\bar{\gamma}_2}} \ (-2 < \bar{\gamma}_2 < 0),$$

where

$$C_6 = \left(\int_{-\infty}^{\infty}\left(-4\bar{\gamma}_2^2x^2 - 8\bar{\gamma}_2^2 - 24\bar{\gamma}_2\right)^{\frac{-5\bar{\gamma}_2-6}{2\bar{\gamma}_2}}dx\right)^{-1}. \tag{A4}$$

In summary, we obtain the symmetric Pearson distribution with the PDF given by

$$f_P(x;\bar{\gamma}_2) = \begin{cases} C_6\left(-4\bar{\gamma}_2^2x^2 - 8\bar{\gamma}_2^2 - 24\bar{\gamma}_2\right)^{\frac{-5\bar{\gamma}_2-6}{2\bar{\gamma}_2}} & (-2 < \bar{\gamma}_2 < 0) \\ \phi(x;0,1) & (\bar{\gamma}_2 = 0) \\ C_4\left(\bar{\gamma}_2x^2 + 2\bar{\gamma}_2 + 6\right)^{\frac{-5\bar{\gamma}_2-6}{2\bar{\gamma}_2}} & (\bar{\gamma}_2 > 0) \end{cases},$$

where C_4 and C_6 are given by (A3) and (A4), respectively.

Tables

Tables A1–A30 present the Monte Carlo simulation results for the alternatives. We assume that a GoFT detects negative or positive $\bar{\gamma}_2$ if its power is at least 0.06. These values are in bold, while the highest PoT values are underlined.

Table A1. $B(8.077, v)$. PoT versus $\bar{\gamma}_2$ and $M(8.077, v; 0, 1), n = 25$

Test	$\bar{\gamma}_2 = -1.2$ $v = 1$ $M = 0.293$	$\bar{\gamma}_2 = -0.6$ $v = 2$ $M = 0.466$	$\bar{\gamma}_2 = -0.4$ $v = 3$ $M = 0.537$	$\bar{\gamma}_2 = -0.3$ $v = 4$ $M = 0.599$	$\bar{\gamma}_2 = -0.2$ $v = 6$ $M = 0.689$	$\bar{\gamma}_2 = -0.15$ $v = 8$ $M = 0.756$	$\bar{\gamma}_2 = -0.1$ $v = 12$ $M = 0.853$	$\bar{\gamma}_2 = -0.055$ $v = 22$ $M = 0.998$
AD	**0.231**	0.037	0.043	0.042	0.047	0.047	0.046	0.049
SW	**0.290**	0.030	0.037	0.036	0.041	0.045	0.047	0.049
KT	**0.280**	0.027	0.025	0.030	0.036	0.039	0.039	0.045
AS	0.005	0.007	0.019	0.023	0.033	0.039	0.044	0.043
SF	**0.123**	0.015	0.025	0.029	0.038	0.044	0.047	0.046
AP	**0.244**	0.018	0.022	0.025	0.033	0.037	0.043	0.044
RJ	**0.112**	0.013	0.023	0.025	0.035	0.040	0.045	0.043
JB	0.002	0.005	0.015	0.020	0.030	0.034	0.042	0.043
H1	**0.290**	0.028	0.036	0.035	0.040	0.044	0.048	0.048
CS	**0.334**	0.035	0.039	0.040	0.043	0.045	0.049	0.050
AJB	0.001	0.005	0.013	0.021	0.032	0.037	0.036	0.046
BS	**0.288**	0.044	0.043	0.042	0.045	0.044	0.047	0.045
ZA	**0.229**	0.021	0.029	0.032	0.039	0.043	0.046	0.047
ZC	**0.333**	0.028	0.030	0.032	0.039	0.042	0.047	0.047
SJ	0.001	0.013	0.024	0.028	0.036	0.036	0.046	0.043
β_3^2	<u>**0.444**</u>	0.041	0.034	0.039	0.041	0.043	0.049	0.046
RJB	0.001	0.005	0.017	0.020	0.032	0.035	0.043	0.043
X_{APD}	**0.260**	0.028	0.033	0.034	0.041	0.042	0.044	0.047
Z_{EPD}	<u>**0.392**</u>	0.052	0.045	0.045	0.047	0.047	0.051	0.048
$LF_{\alpha,\beta}$	**0.166**	0.044	0.055	0.056	0.055	0.058	0.056	0.057

Source: authors' work.

Table A2. $BEP(\sqrt{2}, p, q)$. PoT versus $\bar{\gamma}_2$ and $M(\sqrt{2}, p, q; 0, 1), n = 25$

Test	$\bar{\gamma}_2 = -0.4$ $p = 1.692$ $q = 0.492$ $M = 0.8$	$\bar{\gamma}_2 = -0.3$ $p = 1.745$ $q = 0.358$ $M = 0.85$	$\bar{\gamma}_2 = -0.2$ $p = 1.81$ $q = 0.232$ $M = 0.9$	$\bar{\gamma}_2 = -0.1$ $p = 1.894$ $q = 0.113$ $M = 0.95$	$\bar{\gamma}_2 = 0$ $p = 2$ $q = 0$ $M = 1$	$\bar{\gamma}_2 = 0.05$ $p = 1.874$ $q = 0.043$ $M = 0.975$	$\bar{\gamma}_2 = 0.1$ $p = 1.768$ $q = 0.084$ $M = 0.95$	$\bar{\gamma}_2 = 0.2$ $p = 1.598$ $q = 0.161$ $M = 0.9$	$\bar{\gamma}_2 = 0.3$ $p = 1.465$ $q = 0.233$ $M = 0.85$	$\bar{\gamma}_2 = 0.4$ $p = 1.357$ $q = 0.301$ $M = 0.8$
AD	**0.095**	**0.068**	0.056	0.055	0.051	0.050	0.052	0.059	0.068	**0.071**
SW	**0.077**	**0.061**	0.055	0.052	0.049	0.054	0.057	0.063	**0.076**	**0.081**
KT	0.059	0.046	0.051	0.047	0.048	0.057	0.058	**0.071**	**0.078**	**0.087**
AS	0.032	0.034	0.039	0.045	0.048	0.054	0.060	0.068	**0.083**	**0.085**
SF	0.049	0.046	0.044	0.049	0.051	0.058	0.063	**0.071**	**0.087**	**0.091**
AP	0.053	0.043	0.046	0.047	0.046	0.054	0.060	0.067	**0.083**	**0.090**
RJ	0.046	0.042	0.041	0.047	0.047	0.055	0.060	0.066	**0.082**	**0.087**
JB	0.026	0.029	0.037	0.044	0.047	0.054	0.062	0.068	**0.086**	**0.093**
H1	**0.069**	0.053	0.048	0.050	0.051	0.055	0.054	0.061	**0.072**	**0.074**
CS	**0.087**	**0.066**	0.058	0.054	0.049	0.055	0.058	0.063	**0.075**	**0.080**
AJB	0.024	0.026	0.036	0.042	0.048	0.057	0.059	**0.073**	**0.082**	**0.091**
BS	**0.120**	**0.083**	**0.061**	0.049	0.049	0.048	0.052	0.053	0.059	0.064
ZA	**0.064**	0.055	0.052	0.054	0.046	0.055	0.058	0.067	**0.080**	**0.084**
ZC	**0.070**	0.057	0.053	0.053	0.048	0.054	**0.061**	0.067	**0.080**	**0.084**
SJ	0.014	0.016	0.023	0.036	0.052	0.054	0.054	0.058	0.067	**0.072**
β_3^2	0.060	0.049	0.045	0.046	0.051	0.052	0.055	**0.061**	0.065	**0.072**
RJB	0.019	0.022	0.030	0.041	0.049	0.055	0.059	**0.068**	**0.083**	**0.088**
X_{APD}	**0.084**	**0.061**	0.054	0.051	0.051	0.053	0.059	**0.064**	**0.076**	**0.081**
Z_{EPD}	**0.119**	**0.088**	**0.069**	0.055	0.054	0.053	0.058	0.059	0.068	**0.072**
$LF_{\alpha,\beta}$	**0.124**	**0.093**	**0.072**	0.064	0.059	0.062	0.063	**0.069**	**0.073**	**0.075**

Source: authors' work.

Table A3. $BPN(a)$. PoT versus $\bar{\gamma}_2$ and $M(a; 0, 1), n = 25$

Test	$\bar{\gamma}_2 = -0.4$ $a = 2.059$ $M = 0.912$	$\bar{\gamma}_2 = -0.3$ $a = 1.763$ $M = 0.936$	$\bar{\gamma}_2 = -0.2$ $a = 1.49$ $M = 0.959$	$\bar{\gamma}_2 = -0.1$ $a = 1.236$ $M = 0.98$	$\bar{\gamma}_2 = 0$ $a = 1$ $M = 1$	$\bar{\gamma}_2 = 0.05$ $a = 0.888$ $M = 0.99$	$\bar{\gamma}_2 = 0.1$ $a = 0.779$ $M = 0.981$	$\bar{\gamma}_2 = 0.2$ $a = 0.57$ $M = 0.963$	$\bar{\gamma}_2 = 0.3$ $a = 0.373$ $M = 0.946$	$\bar{\gamma}_2 = 0.4$ $a = 0.186$ $M = 0.93$
AD	**0.061**	0.055	0.049	0.048	0.053	0.050	0.052	**0.064**	**0.068**	**0.073**
SW	0.056	0.052	0.046	0.047	0.054	0.054	0.053	**0.062**	**0.068**	**0.071**
KT	0.045	0.042	0.040	0.046	0.048	0.053	**0.061**	**0.063**	**0.074**	**0.085**
AS	0.027	0.029	0.036	0.044	0.051	0.054	0.055	**0.067**	**0.071**	**0.079**
SF	0.037	0.038	0.040	0.047	0.055	0.057	0.059	**0.074**	**0.081**	**0.087**
AP	0.040	0.040	0.042	0.045	0.052	0.055	0.055	**0.069**	**0.072**	**0.079**
RJ	0.033	0.036	0.036	0.043	0.051	0.054	0.055	**0.069**	**0.077**	**0.083**
JB	0.022	0.026	0.035	0.042	0.051	0.056	0.058	**0.072**	**0.077**	**0.086**
H1	0.051	0.049	0.046	0.044	0.053	0.057	0.056	**0.071**	**0.079**	**0.085**
CS	**0.062**	0.057	0.049	0.049	0.055	0.054	0.051	**0.061**	**0.065**	**0.069**
AJB	0.019	0.027	0.034	0.042	0.046	0.055	**0.063**	**0.066**	**0.081**	**0.095**
BS	**0.074**	**0.062**	0.056	0.049	0.048	0.051	0.053	0.058	**0.073**	**0.073**
ZA	0.047	0.047	0.045	0.048	0.055	0.054	0.052	**0.061**	**0.065**	**0.074**
ZC	0.053	0.050	0.047	0.048	0.056	0.053	0.054	**0.062**	**0.066**	**0.071**
SJ	0.014	0.020	0.029	0.037	0.052	0.057	**0.064**	**0.082**	**0.101**	**0.111**
β_3^2	0.054	0.048	0.048	0.046	0.050	0.055	0.057	**0.068**	**0.077**	**0.083**
RJB	0.019	0.023	0.033	0.041	0.052	0.057	**0.061**	**0.079**	**0.086**	**0.098**
X_{APD}	0.058	0.052	0.047	0.045	0.053	0.054	0.057	**0.066**	**0.074**	**0.080**
Z_{EPD}	**0.083**	**0.070**	**0.062**	0.052	0.050	0.053	0.053	0.058	**0.069**	**0.070**
$LF_{\alpha,\beta}$	**0.079**	**0.065**	0.057	0.058	**0.062**	0.059	**0.063**	**0.069**	**0.077**	**0.080**

Source: authors' work.

Table A4. $BS(a,b)$. PoT versus $\bar{\gamma}_2$ and $M(a,b;0,\sigma)$, $n=25$

Test	$\bar{\gamma}_2=-1$ $a=1.5$ $b=2.41$ $\sigma=3.5$ $M=0.5$	$\bar{\gamma}_2=-0.9$ $a=1.833$ $b=2.431$ $\sigma=3$ $M=0.55$	$\bar{\gamma}_2=-0.8$ $a=2.25$ $b=2.5$ $\sigma=2.5$ $M=0.613$	$\bar{\gamma}_2=-0.7$ $a=2.786$ $b=2.817$ $\sigma=2.388$ $M=0.65$	$\bar{\gamma}_2=-0.6$ $a=3.5$ $b=2.88$ $\sigma=1.985$ $M=0.7$	$\bar{\gamma}_2=-0.5$ $a=4.5$ $b=2$ $\sigma=1.104$ $M=0.75$	$\bar{\gamma}_2=-0.4$ $a=6$ $b=2$ $\sigma=0.853$ $M=0.807$	$\bar{\gamma}_2=-0.3$ $a=8.5$ $b=1.5$ $\sigma=0.494$ $M=0.85$	$\bar{\gamma}_2=-0.2$ $a=13.5$ $b=1.449$ $\sigma=0.227$ $M=0.9$	$\bar{\gamma}_2=-0.1$ $a=28.5$ $b=2.091$ $\sigma=0.307$ $M=0.95$	$\bar{\gamma}_2=-0.036$ $a=81.36$ $b=0.48$ $\sigma=0.038$ $M=0.999$
AD	**0.098**	**0.073**	**0.063**	0.052	0.046	0.046	0.044	0.042	0.044	0.049	0.048
SW	**0.107**	**0.074**	0.055	0.044	0.037	0.042	0.039	0.037	0.044	0.047	0.047
KT	**0.126**	**0.083**	0.058	0.040	0.032	0.028	0.029	0.032	0.039	0.039	0.046
AS	0.004	0.006	0.007	0.007	0.009	0.017	0.020	0.027	0.036	0.040	0.046
SF	0.040	0.027	0.024	0.021	0.020	0.026	0.028	0.031	0.038	0.044	0.051
AP	**0.093**	**0.066**	0.039	0.029	0.023	0.024	0.027	0.028	0.035	0.040	0.045
RJ	0.036	0.024	0.021	0.018	0.018	0.023	0.026	0.029	0.035	0.040	0.047
JB	0.001	0.004	0.004	0.004	0.005	0.013	0.017	0.023	0.032	0.039	0.044
H1	**0.112**	**0.078**	0.051	0.045	0.037	0.037	0.039	0.039	0.043	0.047	0.051
CS	**0.128**	**0.090**	**0.065**	0.053	0.045	0.046	0.043	0.039	0.045	0.047	0.047
AJB	0.001	0.002	0.002	0.003	0.007	0.010	0.014	0.023	0.033	0.039	0.045
BS	**0.151**	**0.115**	**0.088**	**0.068**	0.060	0.050	0.049	0.039	0.044	0.046	0.051
ZA	**0.071**	0.051	0.037	0.032	0.028	0.034	0.032	0.035	0.041	0.045	0.047
ZC	**0.119**	**0.080**	0.054	0.042	0.036	0.039	0.037	0.034	0.043	0.044	0.047
SJ	0.002	0.004	0.005	0.006	0.010	0.015	0.021	0.024	0.032	0.043	0.049
β_3^2	**0.194**	**0.127**	**0.087**	**0.061**	0.049	0.043	0.044	0.037	0.042	0.046	0.052
RJB	0.001	0.003	0.004	0.004	0.006	0.011	0.017	0.025	0.032	0.040	0.046
X_{APD}	**0.114**	**0.079**	0.058	0.046	0.038	0.039	0.038	0.036	0.041	0.045	0.050
Z_{EPD}	<u>**0.212**</u>	<u>**0.156**</u>	<u>**0.119**</u>	<u>**0.087**</u>	<u>**0.071**</u>	0.060	0.055	0.044	0.047	0.050	0.055
$LF_{\alpha,\beta}$	**0.090**	**0.076**	**0.070**	0.059	0.059	0.059	0.057	0.048	0.056	**0.060**	0.057

Source: authors' work.

Table A5. $BSSN(a,b)$. PoT versus $\bar{\gamma}_2$ and $M(a,b;0,\sigma)$, $n=25$

Test	$\bar{\gamma}_2=-1$ $a=1.7$ $b=0.137$ $\sigma=1.8$ $M=0.62$	$\bar{\gamma}_2=-0.9$ $a=1.7$ $b=0.192$ $\sigma=1.7$ $M=0.637$	$\bar{\gamma}_2=-0.8$ $a=1.6$ $b=0.273$ $\sigma=1.6$ $M=0.669$	$\bar{\gamma}_2=-0.7$ $a=1.6$ $b=0.356$ $\sigma=1.5$ $M=0.688$	$\bar{\gamma}_2=-0.6$ $a=1.626$ $b=0.453$ $\sigma=1.41$ $M=0.701$	$\bar{\gamma}_2=-0.5$ $a=2.049$ $b=0.463$ $\sigma=1.092$ $M=0.753$	$\bar{\gamma}_2=-0.4$ $a=2.104$ $b=0.589$ $\sigma=0.939$ $M=0.804$	$\bar{\gamma}_2=-0.3$ $a=2.629$ $b=0.632$ $\sigma=0.711$ $M=0.865$	$\bar{\gamma}_2=-0.2$ $a=3$ $b=0.791$ $\sigma=0.581$ $M=0.909$	$\bar{\gamma}_2=-0.1$ $a=3.033$ $b=1.311$ $\sigma=0.409$ $M=0.95$	$\bar{\gamma}_2=0$ $a=22.241$ $b=13.218$ $\sigma=0.15$ $M=0.999$
AD	**0.261**	**0.183**	**0.129**	**0.090**	**0.082**	0.058	0.048	0.050	0.046	0.049	0.054
SW	**0.219**	**0.155**	**0.107**	**0.079**	**0.069**	0.052	0.044	0.044	0.040	0.047	0.055
KT	**0.224**	**0.143**	**0.107**	**0.068**	0.058	0.039	0.036	0.035	0.039	0.045	0.049
AS	0.009	0.008	0.008	0.010	0.011	0.018	0.023	0.030	0.033	0.043	0.054
SF	**0.105**	**0.074**	0.051	0.036	0.033	0.028	0.029	0.036	0.035	0.047	0.057
AP	**0.182**	**0.118**	**0.083**	0.057	0.051	0.035	0.029	0.035	0.035	0.042	0.052
RJ	**0.097**	**0.068**	0.048	0.032	0.030	0.025	0.026	0.033	0.033	0.045	0.054
JB	0.004	0.004	0.004	0.005	0.006	0.013	0.017	0.026	0.031	0.042	0.052
H1	**0.230**	**0.160**	**0.114**	**0.079**	**0.070**	0.051	0.045	0.045	0.038	0.049	0.057
CS	**0.247**	**0.180**	**0.128**	**0.092**	**0.080**	0.059	0.048	0.047	0.042	0.047	0.056
AJB	0.003	0.002	0.003	0.003	0.005	0.011	0.018	0.026	0.032	0.040	0.053
BS	**0.369**	**0.261**	**0.192**	**0.131**	**0.114**	**0.072**	0.057	0.053	0.041	0.048	0.048
ZA	**0.137**	**0.101**	**0.073**	0.054	0.049	0.040	0.038	0.038	0.039	0.045	0.055
ZC	**0.203**	**0.145**	**0.102**	**0.075**	**0.065**	0.048	0.041	0.042	0.040	0.045	0.054
SJ	0.003	0.004	0.002	0.004	0.006	0.011	0.018	0.026	0.030	0.045	0.052
β_3^2	**0.244**	**0.170**	**0.133**	**0.098**	**0.091**	0.053	0.049	0.045	0.041	0.047	0.052
RJB	0.003	0.004	0.003	0.004	0.006	0.012	0.017	0.027	0.029	0.044	0.052
X_{APD}	**0.291**	**0.198**	**0.136**	**0.093**	**0.081**	0.054	0.043	0.043	0.038	0.046	0.052
Z_{EPD}	<u>**0.407**</u>	<u>**0.294**</u>	<u>**0.221**</u>	<u>**0.158**</u>	<u>**0.139**</u>	<u>**0.086**</u>	<u>**0.068**</u>	0.059	0.046	0.052	0.051
$LF_{\alpha,\beta}$	**0.255**	**0.184**	**0.140**	**0.094**	**0.089**	**0.070**	**0.062**	<u>**0.061**</u>	0.050	0.056	<u>**0.063**</u>

Source: authors' work.

Table A6. $CH(2,b)$. PoT versus $\bar{\gamma}_2$ and $M(2,b;0,\sigma), n = 25$

Test	$\bar{\gamma}_2 = -0.4$ $b = 42.516$ $\sigma = 1.127$ $M = 0.8$	$\bar{\gamma}_2 = -0.3$ $b = 28.874$ $\sigma = 1.161$ $M = 0.85$	$\bar{\gamma}_2 = -0.2$ $b = 20.586$ $\sigma = 1.204$ $M = 0.9$	$\bar{\gamma}_2 = -0.1$ $b = 15.225$ $\sigma = 1.258$ $M = 0.95$	$\bar{\gamma}_2 = 0$ $b = 11.592$ $\sigma = 1.28$ $M = 0.975$	$\bar{\gamma}_2 = 0.05$ $b = 10.202$ $\sigma = 1.156$ $M = 0.95$	$\bar{\gamma}_2 = 0.1$ $b = 9.023$ $\sigma = 1.071$ $M = 0.926$	$\bar{\gamma}_2 = 0.2$ $b = 7.149$ $\sigma = 1.405$ $M = 0.914$	$\bar{\gamma}_2 = 0.3$ $b = 5.743$ $\sigma = 1.536$ $M = 0.852$	$\bar{\gamma}_2 = 0.4$ $b = 4.665$ $\sigma = 1.645$ $M = 0.802$
AD	0.055	0.057	0.055	0.049	0.053	0.052	0.058	0.053	0.058	**0.062**
SW	0.051	0.049	0.052	0.051	0.054	0.056	**0.062**	0.058	**0.066**	**0.071**
KT	0.060	0.050	0.047	0.050	0.050	0.048	**0.064**	**0.061**	**0.068**	**0.069**
AS	0.020	0.025	0.036	0.041	0.049	0.052	0.058	0.059	**0.075**	**0.081**
SF	0.030	0.032	0.041	0.045	0.049	0.055	**0.064**	**0.062**	**0.075**	**0.082**
AP	0.034	0.042	0.045	0.047	0.053	0.054	**0.061**	**0.063**	**0.077**	**0.083**
RJ	0.028	0.029	0.037	0.043	0.046	0.051	0.059	0.059	**0.071**	**0.077**
JB	0.015	0.022	0.031	0.038	0.047	0.050	0.058	**0.064**	**0.077**	**0.086**
H1	0.047	0.049	0.049	0.048	0.053	0.051	0.059	0.055	**0.062**	**0.070**
CS	0.056	0.056	0.058	0.055	0.058	0.058	**0.065**	**0.062**	**0.067**	**0.071**
AJB	0.015	0.022	0.028	0.036	0.041	0.050	**0.064**	0.057	**0.065**	**0.063**
BS	**0.072**	**0.067**	**0.062**	0.052	0.052	0.053	0.055	0.054	0.059	**0.065**
ZA	0.041	0.044	0.047	0.049	0.055	0.057	**0.062**	**0.062**	**0.072**	**0.076**
ZC	0.047	0.050	0.054	0.053	0.057	0.058	**0.064**	**0.063**	**0.072**	**0.077**
SJ	0.013	0.018	0.022	0.029	0.035	0.038	0.047	0.050	**0.064**	**0.077**
β_3^2	0.057	0.057	0.050	0.050	0.050	0.051	0.054	0.056	**0.064**	**0.071**
RJB	0.014	0.019	0.027	0.033	0.043	0.047	0.054	0.058	**0.075**	**0.085**
X_{APD}	0.051	0.051	0.052	0.049	0.054	0.055	**0.061**	0.060	**0.070**	**0.075**
Z_{EPD}	**0.085**	**0.077**	**0.075**	**0.061**	**0.064**	0.059	**0.063**	**0.064**	**0.068**	**0.073**
$LF_{\alpha,\beta}$	**0.067**	**0.066**	**0.066**	**0.062**	**0.062**	0.060	**0.066**	**0.062**	**0.066**	**0.072**

Source: authors' work.

Table A7. $DSN(a,b)$. PoT versus $\bar{\gamma}_2$ and $M(a,b;0,1), n = 25$

Test	$\bar{\gamma}_2 = -1$ $a = 3.315$ $b = 2.094$ $M = 0.5$	$\bar{\gamma}_2 = -0.9$ $a = 1.924$ $b = 2.019$ $M = 0.55$	$\bar{\gamma}_2 = -0.8$ $a = 1.112$ $b = 1.924$ $M = 0.6$	$\bar{\gamma}_2 = -0.7$ $a = 0.634$ $b = 1.816$ $M = 0.65$	$\bar{\gamma}_2 = -0.6$ $a = 0.353$ $b = 1.698$ $M = 0.7$	$\bar{\gamma}_2 = -0.5$ $a = 0.191$ $b = 1.575$ $M = 0.75$	$\bar{\gamma}_2 = -0.4$ $a = 0.098$ $b = 1.451$ $M = 0.8$	$\bar{\gamma}_2 = -0.3$ $a = 0.047$ $b = 1.33$ $M = 0.85$	$\bar{\gamma}_2 = -0.2$ $a = 0.02$ $b = 1.213$ $M = 0.9$	$\bar{\gamma}_2 = -0.1$ $a = 0.009$ $b = 1.221$ $M = 0.95$	$\bar{\gamma}_2 = 0$ $a = 0$ $b = 1$ $M = 1$
AD	**0.138**	**0.102**	**0.074**	0.057	0.052	0.043	0.043	0.048	0.045	0.048	0.046
SW	**0.132**	**0.100**	**0.068**	0.051	0.048	0.041	0.039	0.040	0.039	0.045	0.047
KT	**0.148**	**0.100**	**0.072**	0.052	0.037	0.032	0.031	0.032	0.036	0.038	0.048
AS	0.008	0.008	0.007	0.008	0.013	0.017	0.020	0.028	0.034	0.040	0.047
SF	0.056	0.040	0.027	0.024	0.027	0.025	0.027	0.034	0.038	0.042	0.051
AP	**0.113**	**0.079**	0.054	0.037	0.033	0.028	0.026	0.029	0.035	0.042	0.048
RJ	0.051	0.036	0.024	0.021	0.023	0.023	0.025	0.031	0.036	0.039	0.048
JB	0.004	0.003	0.004	0.005	0.010	0.013	0.016	0.023	0.032	0.039	0.048
H1	**0.139**	**0.101**	**0.072**	0.052	0.045	0.041	0.037	0.041	0.042	0.044	0.050
CS	**0.157**	**0.116**	**0.082**	**0.062**	0.056	0.046	0.042	0.044	0.041	0.046	0.049
AJB	0.001	0.002	0.003	0.005	0.006	0.011	0.015	0.022	0.031	0.037	0.047
BS	**0.198**	**0.153**	**0.114**	**0.088**	**0.067**	0.057	0.047	0.042	0.047	0.041	0.051
ZA	**0.092**	**0.065**	0.047	0.038	0.038	0.033	0.034	0.037	0.036	0.043	0.048
ZC	**0.138**	**0.101**	**0.070**	0.050	0.045	0.039	0.037	0.037	0.036	0.042	0.049
SJ	0.002	0.002	0.005	0.007	0.010	0.013	0.020	0.027	0.036	0.039	0.047
β_3^2	**0.195**	**0.140**	**0.105**	**0.076**	0.059	0.048	0.043	0.041	0.042	0.043	0.049
RJB	0.002	0.002	0.004	0.005	0.009	0.012	0.017	0.024	0.032	0.038	0.046
X_{APD}	**0.155**	**0.112**	**0.079**	0.056	0.048	0.042	0.038	0.037	0.040	0.043	0.049
Z_{EPD}	**0.256**	**0.200**	**0.148**	**0.112**	**0.084**	**0.069**	0.055	0.048	0.050	0.047	0.051
$LF_{\alpha,\beta}$	**0.133**	**0.099**	**0.080**	**0.069**	**0.064**	0.057	0.055	0.057	0.058	0.052	0.055

Source: authors' work.

Table A8. $ECK(8.326, p)$. PoT versus $\bar{\gamma}_2$ and $M(8.326, p; 0, \sigma)$, $n = 25$

Test	$\bar{\gamma}_2 = -1$ $p = 0.5$ $\sigma = 1.624$ $M = 0.5$	$\bar{\gamma}_2 = -0.9$ $p = 0.833$ $\sigma = 1.66$ $M = 0.55$	$\bar{\gamma}_2 = -0.8$ $p = 1.25$ $\sigma = 1.684$ $M = 0.6$	$\bar{\gamma}_2 = -0.7$ $p = 1.786$ $\sigma = 1.694$ $M = 0.65$	$\bar{\gamma}_2 = -0.6$ $p = 2.5$ $\sigma = 1.687$ $M = 0.7$	$\bar{\gamma}_2 = -0.5$ $p = 3.5$ $\sigma = 1.655$ $M = 0.75$	$\bar{\gamma}_2 = -0.4$ $p = 5$ $\sigma = 1.592$ $M = 0.8$	$\bar{\gamma}_2 = -0.3$ $p = 7.5$ $\sigma = 1.484$ $M = 0.85$	$\bar{\gamma}_2 = -0.2$ $p = 12.5$ $\sigma = 1.305$ $M = 0.9$	$\bar{\gamma}_2 = -0.1$ $p = 27.5$ $\sigma = 0.995$ $M = 0.95$	$\bar{\gamma}_2 = -0.018$ $p = 160.706$ $\sigma = 0.463$ $M = 0.999$
AD	**0.099**	**0.077**	**0.060**	0.048	0.048	0.041	0.042	0.048	0.045	0.048	0.045
SW	**0.108**	**0.079**	0.055	0.042	0.043	0.038	0.037	0.040	0.039	0.045	0.046
KT	**0.123**	**0.081**	0.056	0.043	0.031	0.029	0.028	0.031	0.035	0.038	0.047
AS	0.006	0.006	0.004	0.007	0.011	0.014	0.020	0.027	0.033	0.041	0.046
SF	0.039	0.029	0.021	0.019	0.023	0.022	0.026	0.034	0.038	0.042	0.050
AP	**0.091**	0.060	0.041	0.029	0.027	0.024	0.025	0.028	0.034	0.041	0.047
RJ	0.034	0.026	0.018	0.017	0.021	0.021	0.025	0.031	0.036	0.039	0.047
JB	0.002	0.002	0.003	0.005	0.008	0.011	0.015	0.022	0.031	0.040	0.046
H1	**0.108**	**0.077**	0.053	0.041	0.039	0.038	0.036	0.041	0.041	0.043	0.049
CS	**0.128**	**0.095**	**0.066**	0.051	0.048	0.042	0.040	0.043	0.040	0.046	0.047
AJB	0.000	0.001	0.003	0.004	0.006	0.010	0.014	0.021	0.030	0.037	0.045
BS	**0.145**	**0.117**	**0.088**	**0.071**	0.058	0.051	0.044	0.041	0.046	0.041	0.051
ZA	**0.073**	0.050	0.035	0.031	0.032	0.029	0.032	0.036	0.036	0.043	0.047
ZC	**0.117**	**0.084**	0.055	0.042	0.039	0.035	0.035	0.036	0.035	0.043	0.048
SJ	0.002	0.003	0.005	0.008	0.011	0.014	0.021	0.027	0.036	0.039	0.046
β_3^2	**0.184**	**0.126**	**0.090**	**0.066**	0.051	0.044	0.039	0.041	0.042	0.043	0.049
RJB	0.002	0.002	0.003	0.005	0.009	0.012	0.016	0.023	0.032	0.038	0.045
X_{APD}	**0.109**	**0.081**	0.059	0.045	0.040	0.038	0.035	0.036	0.040	0.043	0.047
Z_{EPD}	**0.202**	**0.157**	**0.116**	**0.092**	**0.073**	**0.061**	0.053	0.046	0.049	0.047	0.050
$LF_{\alpha,\beta}$	**0.092**	**0.076**	**0.064**	**0.060**	0.059	0.055	0.054	0.056	0.058	0.052	0.054

Source: authors' work.

Table A9. $ES(\bar{\gamma}_2)$. PoT versus $\bar{\gamma}_2$ and $M(\bar{\gamma}_2; 0,1)$, $n = 25$

Test	$\bar{\gamma}_2 = -1$ $M = 0.942$	$\bar{\gamma}_2 = -0.9$ $M = 0.947$	$\bar{\gamma}_2 = -0.8$ $M = 0.953$	$\bar{\gamma}_2 = -0.7$ $M = 0.959$	$\bar{\gamma}_2 = -0.6$ $M = 0.965$	$\bar{\gamma}_2 = 0$ $M = 1$	$\bar{\gamma}_2 = 0.1$ $M = 0.994$	$\bar{\gamma}_2 = 0.2$ $M = 0.988$	$\bar{\gamma}_2 = 0.3$ $M = 0.982$	$\bar{\gamma}_2 = 0.4$ $M = 0.977$	$\bar{\gamma}_2 = 0.5$ $M = 0.971$
AD	0.047	0.046	0.045	0.045	0.046	0.051	0.053	0.056	**0.063**	**0.067**	**0.072**
SW	0.041	0.037	0.039	0.038	0.039	0.047	0.055	0.058	**0.064**	**0.077**	**0.080**
KT	0.032	0.030	0.028	0.025	0.027	0.046	**0.061**	**0.061**	**0.075**	**0.086**	**0.094**
AS	0.014	0.016	0.017	0.017	0.020	0.045	0.054	**0.063**	**0.072**	**0.082**	**0.090**
SF	0.025	0.023	0.024	0.024	0.026	0.049	0.059	**0.068**	**0.078**	**0.089**	**0.098**
AP	0.023	0.025	0.026	0.024	0.028	0.048	0.056	**0.065**	**0.073**	**0.083**	**0.096**
RJ	0.022	0.021	0.022	0.022	0.024	0.046	0.054	**0.064**	**0.074**	**0.084**	**0.093**
JB	0.009	0.011	0.013	0.012	0.016	0.047	0.054	**0.068**	**0.078**	**0.090**	**0.104**
H1	0.037	0.037	0.038	0.038	0.035	0.052	0.055	**0.062**	**0.071**	**0.077**	**0.083**
CS	0.046	0.044	0.044	0.042	0.043	0.049	0.055	0.058	**0.064**	**0.074**	**0.077**
AJB	0.008	0.008	0.009	0.013	0.014	0.047	**0.063**	**0.064**	**0.080**	**0.091**	**0.102**
BS	0.058	0.055	0.051	0.053	0.047	0.047	0.050	0.059	0.059	**0.065**	**0.068**
ZA	0.032	0.031	0.033	0.032	0.034	0.048	0.053	0.059	**0.069**	**0.080**	**0.086**
ZC	0.037	0.036	0.037	0.036	0.037	0.049	0.055	0.060	**0.068**	**0.079**	**0.082**
SJ	0.013	0.014	0.016	0.018	0.019	0.048	0.056	**0.066**	**0.078**	**0.089**	**0.093**
β_3^2	0.045	0.046	0.045	0.043	0.040	0.050	0.054	**0.063**	**0.068**	**0.077**	**0.086**
RJB	0.009	0.012	0.012	0.012	0.015	0.047	0.056	**0.069**	**0.084**	**0.093**	**0.101**
X_{APD}	0.041	0.036	0.038	0.038	0.034	0.049	0.056	**0.064**	**0.071**	**0.082**	**0.085**
Z_{EPD}	**0.068**	**0.066**	**0.062**	0.060	0.054	0.050	0.051	0.060	**0.063**	**0.072**	**0.076**
$LF_{\alpha,\beta}$	0.057	0.058	0.058	0.054	0.055	**0.062**	0.060	**0.064**	**0.070**	**0.074**	**0.071**

Source: authors' work.

Table A10. $EP(a)$. PoT versus $\bar{\gamma}_2$ and $M(a;0,\sigma)$, $n=25$

Test	$\bar{\gamma}_2=-0.7$ $a=3.422$ $\sigma=1.817$ $M=0.65$	$\bar{\gamma}_2=-0.6$ $a=3.057$ $\sigma=1.682$ $M=0.7$	$\bar{\gamma}_2=-0.5$ $a=2.779$ $\sigma=1.552$ $M=0.75$	$\bar{\gamma}_2=-0.4$ $a=2.559$ $\sigma=1.428$ $M=0.8$	$\bar{\gamma}_2=-0.3$ $a=2.381$ $\sigma=1.311$ $M=0.85$	$\bar{\gamma}_2=0$ $a=2$ $\sigma=1$ $M=1$	$\bar{\gamma}_2=0.1$ $a=1.907$ $\sigma=1.118$ $M=0.95$	$\bar{\gamma}_2=0.2$ $a=1.824$ $\sigma=1.253$ $M=0.9$	$\bar{\gamma}_2=0.3$ $a=1.752$ $\sigma=1.406$ $M=0.85$	$\bar{\gamma}_2=0.4$ $a=1.687$ $\sigma=1.584$ $M=0.8$	$\bar{\gamma}_2=0.5$ $a=1.628$ $\sigma=1.79$ $M=0.75$
AD	**0.064**	0.056	0.051	0.049	0.048	0.047	0.054	0.057	**0.061**	**0.068**	**0.077**
SW	0.055	0.049	0.046	0.046	0.044	0.051	0.059	**0.063**	**0.065**	**0.072**	**0.082**
KT	0.050	0.043	0.039	0.035	0.037	0.053	0.060	**0.067**	**0.071**	**0.087**	**0.095**
AS	0.011	0.014	0.017	0.024	0.032	0.053	0.060	**0.070**	**0.075**	**0.084**	**0.092**
SF	0.026	0.025	0.029	0.031	0.036	0.053	**0.062**	**0.071**	**0.078**	**0.086**	**0.100**
AP	0.040	0.033	0.031	0.033	0.032	0.052	**0.061**	**0.069**	**0.079**	**0.085**	**0.098**
RJ	0.024	0.021	0.026	0.028	0.032	0.049	0.059	**0.067**	**0.074**	**0.082**	**0.095**
JB	0.006	0.009	0.016	0.021	0.025	0.053	**0.063**	**0.072**	**0.083**	**0.091**	**0.104**
H1	0.056	0.049	0.044	0.043	0.042	0.052	**0.061**	**0.063**	**0.070**	**0.084**	**0.093**
CS	**0.063**	0.057	0.052	0.050	0.047	0.052	0.058	**0.062**	**0.063**	**0.069**	**0.078**
AJB	0.005	0.006	0.012	0.019	0.025	0.052	**0.062**	**0.074**	**0.077**	**0.093**	**0.104**
BS	**0.090**	**0.071**	**0.069**	0.053	0.048	0.047	0.055	0.058	**0.063**	**0.068**	**0.078**
ZA	0.042	0.038	0.037	0.041	0.040	0.050	0.060	**0.064**	**0.067**	**0.074**	**0.084**
ZC	0.051	0.047	0.043	0.044	0.044	0.052	0.057	**0.065**	**0.067**	**0.073**	**0.083**
SJ	0.006	0.008	0.016	0.018	0.024	0.050	**0.062**	**0.073**	**0.088**	**0.098**	**0.112**
β_3^2	**0.074**	0.060	0.054	0.047	0.042	0.048	0.057	**0.065**	**0.072**	**0.080**	**0.089**
RJB	0.006	0.008	0.015	0.018	0.024	0.053	**0.065**	**0.071**	**0.089**	**0.098**	**0.110**
X_{APD}	**0.062**	0.052	0.048	0.044	0.041	0.051	0.059	**0.063**	**0.071**	**0.081**	**0.093**
Z_{EPD}	**0.111**	**0.093**	**0.078**	**0.062**	0.052	0.050	0.056	0.059	**0.065**	**0.069**	**0.079**
$LF_{\alpha,\beta}$	**0.071**	**0.066**	**0.066**	0.058	0.060	0.055	**0.068**	**0.064**	**0.069**	**0.069**	**0.077**

Source: authors' work.

Table A11. $EN(a)$. PoT versus $\bar{\gamma}_2$ and $M(a;0,1)$, $n=25$

Test	$\bar{\gamma}_2=-1$ $a=2.155$ $M=0.669$	$\bar{\gamma}_2=-0.9$ $a=1.535$ $M=0.707$	$\bar{\gamma}_2=-0.8$ $a=1.145$ $M=0.742$	$\bar{\gamma}_2=-0.7$ $a=0.877$ $M=0.774$	$\bar{\gamma}_2=-0.6$ $a=0.679$ $M=0.804$	$\bar{\gamma}_2=-0.5$ $a=0.527$ $M=0.833$	$\bar{\gamma}_2=-0.4$ $a=0.404$ $M=0.861$	$\bar{\gamma}_2=-0.3$ $a=0.301$ $M=0.888$	$\bar{\gamma}_2=-0.2$ $a=0.211$ $M=0.916$	$\bar{\gamma}_2=-0.1$ $a=0.126$ $M=0.946$	$\bar{\gamma}_2=0$ $a=0$ $M=1$
AD	**0.143**	**0.125**	**0.108**	**0.088**	**0.071**	**0.062**	0.047	0.047	0.047	0.046	0.049
SW	**0.144**	**0.119**	**0.102**	**0.083**	0.060	0.057	0.049	0.042	0.042	0.044	0.048
KT	**0.157**	**0.134**	**0.100**	**0.076**	0.056	0.043	0.034	0.034	0.039	0.041	0.051
AS	0.007	0.007	0.010	0.012	0.016	0.023	0.026	0.029	0.034	0.042	0.051
SF	**0.061**	0.050	0.043	0.038	0.032	0.033	0.031	0.032	0.038	0.043	0.053
AP	**0.125**	**0.100**	**0.077**	**0.062**	0.043	0.039	0.036	0.032	0.038	0.042	0.051
RJ	0.057	0.043	0.038	0.034	0.029	0.031	0.029	0.030	0.035	0.041	0.051
JB	0.003	0.003	0.004	0.008	0.010	0.018	0.020	0.026	0.033	0.040	0.053
H1	**0.160**	**0.130**	**0.108**	**0.086**	0.058	0.053	0.044	0.041	0.043	0.045	0.050
CS	**0.173**	**0.141**	**0.119**	**0.096**	**0.069**	**0.063**	0.053	0.045	0.045	0.046	0.047
AJB	0.002	0.003	0.003	0.005	0.008	0.013	0.018	0.024	0.032	0.039	0.050
BS	**0.199**	**0.179**	**0.152**	**0.127**	**0.095**	**0.076**	0.056	0.050	0.045	0.047	0.049
ZA	**0.104**	**0.083**	**0.072**	**0.060**	0.046	0.047	0.044	0.038	0.040	0.043	0.051
ZC	**0.153**	**0.122**	**0.102**	**0.079**	0.056	0.053	0.047	0.040	0.042	0.044	0.051
SJ	0.002	0.003	0.004	0.005	0.007	0.013	0.017	0.026	0.032	0.044	0.050
β_3^2	**0.213**	**0.172**	**0.137**	**0.103**	**0.068**	0.056	0.048	0.041	0.046	0.045	0.049
RJB	0.002	0.003	0.004	0.006	0.009	0.015	0.018	0.025	0.030	0.039	0.053
X_{APD}	**0.159**	**0.136**	**0.113**	**0.094**	**0.066**	0.059	0.043	0.039	0.043	0.043	0.052
Z_{EPD}	**0.257**	**0.227**	**0.194**	**0.155**	**0.111**	**0.089**	**0.066**	0.057	0.053	0.049	0.054
$LF_{\alpha,\beta}$	**0.128**	**0.115**	**0.108**	**0.097**	**0.082**	**0.068**	0.059	0.058	0.056	0.060	0.059

Source: authors' work.

Table A12. $EL(a)$. PoT versus $\bar{\gamma}_2$ and $M(a;0,\sigma)$, $n = 25$

Test	$\bar{\gamma}_2 = 0.5$ $a = 10.164$ $\sigma = 2.933$ $M = 0.85$	$\bar{\gamma}_2 = 0.6$ $a = 6.162$ $\sigma = 3.312$ $M = 0.8$	$\bar{\gamma}_2 = 0.7$ $a = 4.342$ $\sigma = 3.685$ $M = 0.75$	$\bar{\gamma}_2 = 0.8$ $a = 3.302$ $\sigma = 4.063$ $M = 0.7$	$\bar{\gamma}_2 = 0.9$ $a = 2.627$ $\sigma = 0.968$ $M = 0.65$	$\bar{\gamma}_2 = 1$ $a = 2.155$ $\sigma = 0.83$ $M = 0.6$	$\bar{\gamma}_2 = 1.1$ $a = 1.805$ $\sigma = 0.705$ $M = 0.55$	$\bar{\gamma}_2 = 1.2$ $a = 1.535$ $\sigma = 0.593$ $M = 0.5$	$\bar{\gamma}_2 = 1.3$ $a = 1.32$ $\sigma = 0.493$ $M = 0.45$	$\bar{\gamma}_2 = 1.4$ $a = 1.146$ $\sigma = 0.403$ $M = 0.4$	$\bar{\gamma}_2 = 1.5$ $a = 1$ $\sigma = 0.323$ $M = 0.35$
AD	**0.065**	0.058	0.058	0.057	0.060	**0.062**	**0.075**	**0.077**	**0.090**	0.059	**0.112**
SW	0.056	0.056	0.058	0.057	**0.064**	**0.062**	**0.074**	**0.081**	**0.092**	0.058	**0.116**
KT	0.041	0.042	0.052	0.057	**0.069**	**0.079**	**0.087**	**0.096**	**0.114**	0.050	**0.135**
AS	0.042	0.052	0.055	**0.063**	**0.069**	**0.072**	**0.084**	**0.094**	**0.100**	0.056	**0.123**
SF	0.050	0.056	0.060	**0.065**	**0.074**	**0.078**	**0.093**	**0.107**	**0.117**	0.060	**0.147**
AP	0.041	0.050	0.052	**0.061**	**0.068**	**0.074**	**0.082**	**0.097**	**0.105**	0.056	**0.129**
RJ	0.046	0.053	0.055	**0.061**	**0.070**	**0.073**	**0.088**	**0.100**	**0.111**	0.055	**0.141**
JB	0.038	0.049	0.053	**0.063**	**0.074**	**0.080**	**0.092**	**0.109**	**0.119**	0.053	**0.145**
H1	0.050	0.052	0.055	0.057	**0.066**	**0.072**	**0.088**	**0.097**	**0.115**	0.057	**0.143**
CS	0.059	0.057	0.059	0.056	**0.063**	**0.060**	**0.072**	**0.078**	**0.087**	0.059	**0.110**
AJB	0.034	0.040	0.054	**0.063**	**0.074**	**0.088**	**0.094**	**0.109**	**0.123**	0.047	**0.147**
BS	0.053	0.049	0.048	0.049	0.054	0.057	**0.066**	**0.080**	**0.092**	0.050	**0.119**
ZA	0.054	0.057	**0.060**	0.059	**0.067**	**0.064**	**0.078**	**0.083**	**0.091**	0.060	**0.117**
ZC	0.053	0.056	0.056	0.058	**0.064**	**0.063**	**0.074**	**0.081**	**0.088**	0.058	**0.113**
SJ	0.026	0.037	0.043	0.054	**0.069**	**0.080**	**0.100**	**0.119**	**0.140**	0.041	**0.176**
β_3^2	0.037	0.045	0.046	0.054	**0.067**	**0.075**	**0.085**	**0.099**	**0.112**	0.049	**0.140**
RJB	0.032	0.042	0.048	**0.060**	**0.072**	**0.085**	**0.097**	**0.117**	**0.131**	0.048	**0.166**
X_{APD}	0.052	0.053	0.053	0.055	**0.062**	**0.071**	**0.082**	**0.096**	**0.107**	0.056	**0.136**
Z_{EPD}	0.052	0.054	0.049	0.049	0.055	**0.061**	**0.067**	**0.081**	**0.091**	0.055	**0.112**
$LF_{\alpha,\beta}$	**0.084**	**0.072**	**0.071**	**0.066**	**0.069**	**0.073**	**0.076**	**0.077**	**0.089**	**0.067**	**0.112**

Source: authors' work.

Table A13. $ET(a,v)$. PoT versus $\bar{\gamma}_2$ and $M(a,v;0,\sigma)$, $n = 25$

Test	$\bar{\gamma}_2 = -0.5$ $a = 0.86$ $v = 28$ $\sigma = 0.956$ $M = 0.75$	$\bar{\gamma}_2 = -0.4$ $a = 0.693$ $v = 27$ $\sigma = 1.007$ $M = 0.8$	$\bar{\gamma}_2 = -0.3$ $a = 0.576$ $v = 25$ $\sigma = 1.071$ $M = 0.85$	$\bar{\gamma}_2 = -0.2$ $a = 0.479$ $v = 23.268$ $\sigma = 1.141$ $M = 0.9$	$\bar{\gamma}_2 = -0.1$ $a = 0.452$ $v = 18.978$ $\sigma = 1.259$ $M = 0.95$	$\bar{\gamma}_2 = 0$ $a = 0.071$ $v = 149.141$ $\sigma = 1.072$ $M = 1$	$\bar{\gamma}_2 = 0.1$ $a = 0.423$ $v = 14$ $\sigma = 1.248$ $M = 0.95$	$\bar{\gamma}_2 = 0.2$ $a = 0.492$ $v = 11.5$ $\sigma = 1.166$ $M = 0.9$	$\bar{\gamma}_2 = 0.3$ $a = 0.562$ $v = 10$ $\sigma = 1.087$ $M = 0.85$	$\bar{\gamma}_2 = 0.4$ $a = 0.632$ $v = 9$ $\sigma = 1.009$ $M = 0.8$	$\bar{\gamma}_2 = 0.5$ $a = 1.482$ $v = 7.041$ $\sigma = 1.118$ $M = 0.75$
AD	**0.074**	**0.065**	0.057	0.052	0.049	0.054	0.051	0.057	0.058	0.059	**0.084**
SW	**0.069**	0.059	0.053	0.052	0.051	0.053	0.059	**0.060**	**0.067**	**0.068**	**0.077**
KT	0.060	0.051	0.045	0.043	0.049	0.046	**0.066**	**0.073**	**0.070**	**0.071**	**0.067**
AS	0.021	0.026	0.031	0.040	0.043	0.052	0.059	**0.061**	**0.065**	**0.064**	0.046
SF	0.039	0.037	0.038	0.044	0.045	0.056	0.058	**0.062**	**0.066**	**0.067**	**0.060**
AP	0.051	0.046	0.043	0.045	0.050	0.052	**0.061**	**0.064**	**0.070**	**0.069**	0.058
RJ	0.036	0.035	0.034	0.041	0.042	0.052	0.055	0.058	**0.062**	**0.062**	0.055
JB	0.016	0.022	0.027	0.035	0.042	0.051	0.057	**0.061**	**0.066**	**0.063**	0.039
H1	**0.068**	0.055	0.048	0.049	0.046	0.054	0.053	0.058	**0.063**	0.059	**0.070**
CS	**0.079**	**0.066**	0.058	0.055	0.053	0.054	**0.061**	**0.062**	**0.068**	**0.070**	**0.083**
AJB	0.016	0.023	0.027	0.032	0.046	0.048	**0.061**	**0.068**	**0.067**	**0.067**	0.039
BS	**0.098**	**0.079**	**0.062**	0.051	0.052	0.053	0.050	0.056	0.059	0.058	**0.091**
ZA	0.055	0.049	0.047	0.048	0.051	0.053	**0.060**	**0.064**	**0.069**	**0.069**	**0.071**
ZC	**0.065**	0.056	0.052	0.053	0.051	0.053	**0.062**	**0.065**	**0.071**	**0.072**	**0.075**
SJ	0.007	0.013	0.017	0.026	0.030	0.052	0.043	0.046	0.044	0.042	0.019
β_3^2	**0.075**	**0.062**	0.051	0.049	0.046	0.052	0.051	0.056	0.056	0.058	0.060
RJB	0.013	0.017	0.023	0.032	0.038	0.050	0.052	0.055	**0.061**	0.055	0.031
X_{APD}	**0.073**	**0.063**	0.053	0.048	0.049	0.055	**0.060**	**0.061**	**0.067**	**0.062**	**0.077**
Z_{EPD}	**0.117**	**0.097**	**0.076**	**0.063**	0.060	0.055	**0.060**	**0.066**	**0.070**	**0.071**	**0.100**
$LF_{\alpha,\beta}$	**0.081**	**0.075**	**0.065**	**0.065**	**0.063**	**0.061**	**0.061**	**0.064**	**0.068**	**0.067**	**0.103**

Source: authors' work.

Table A14. $GE(a)$. PoT versus $\bar{\gamma}_2$ and $M(a;0,\sigma)$, $n=25$

Test	$\bar{\gamma}_2=-0.5$ $a=0.36$ $\sigma=1.379$ $M=0.75$	$\bar{\gamma}_2=-0.4$ $a=0.391$ $\sigma=1.297$ $M=0.8$	$\bar{\gamma}_2=-0.3$ $a=0.42$ $\sigma=0.668$ $M=0.85$	$\bar{\gamma}_2=-0.2$ $a=0.448$ $\sigma=0.764$ $M=0.9$	$\bar{\gamma}_2=-0.1$ $a=0.474$ $\sigma=0.873$ $M=0.95$	$\bar{\gamma}_2=0$ $a=0.5$ $\sigma=1$ $M=1$	$\bar{\gamma}_2=0.05$ $a=0.512$ $\sigma=0.966$ $M=0.975$	$\bar{\gamma}_2=0.1$ $a=0.525$ $\sigma=0.932$ $M=0.95$	$\bar{\gamma}_2=0.2$ $a=0.548$ $\sigma=0.866$ $M=0.9$	$\bar{\gamma}_2=0.3$ $a=0.571$ $\sigma=0.802$ $M=0.85$	$\bar{\gamma}_2=0.4$ $a=0.593$ $\sigma=0.741$ $M=0.8$	$\bar{\gamma}_2=0.5$ $a=0.614$ $\sigma=0.681$ $M=0.75$
AD	0.047	0.045	0.045	0.046	0.048	0.053	0.054	0.054	0.056	**0.063**	**0.065**	**0.071**
SW	0.042	0.041	0.041	0.044	0.046	0.053	0.055	0.058	0.058	**0.064**	**0.067**	**0.076**
KT	0.038	0.035	0.039	0.039	0.042	0.053	0.056	0.059	**0.068**	**0.076**	**0.089**	**0.091**
AS	0.018	0.022	0.030	0.038	0.042	0.056	0.053	**0.063**	**0.067**	**0.076**	**0.080**	**0.084**
SF	0.025	0.031	0.034	0.040	0.044	0.059	0.057	**0.064**	**0.068**	**0.080**	**0.082**	**0.093**
AP	0.030	0.031	0.034	0.040	0.042	0.055	0.052	**0.065**	**0.067**	**0.076**	**0.081**	**0.083**
RJ	0.023	0.028	0.031	0.036	0.042	0.055	0.053	**0.060**	**0.064**	**0.076**	**0.077**	**0.088**
JB	0.014	0.018	0.026	0.035	0.041	0.055	0.054	**0.066**	**0.073**	**0.082**	**0.089**	**0.092**
H1	0.040	0.041	0.042	0.044	0.046	0.056	0.057	**0.060**	0.060	**0.076**	**0.076**	**0.085**
CS	0.047	0.045	0.045	0.047	0.048	0.055	0.057	0.058	0.057	**0.063**	**0.066**	**0.074**
AJB	0.012	0.017	0.027	0.032	0.040	0.053	0.057	**0.062**	**0.074**	**0.082**	**0.095**	**0.103**
BS	**0.060**	0.054	0.052	0.047	0.045	0.053	0.051	0.055	0.057	**0.066**	**0.068**	**0.075**
ZA	0.036	0.038	0.039	0.043	0.043	0.055	0.055	**0.061**	**0.060**	**0.069**	**0.072**	**0.079**
ZC	0.040	0.040	0.042	0.043	0.046	0.056	0.055	**0.061**	**0.060**	**0.066**	**0.071**	**0.074**
SJ	0.012	0.019	0.025	0.029	0.042	0.055	0.054	**0.066**	**0.072**	**0.092**	**0.093**	**0.103**
β_3^2	0.046	0.046	0.045	0.042	0.046	0.052	0.051	**0.061**	**0.064**	**0.073**	**0.078**	**0.084**
RJB	0.012	0.018	0.026	0.032	0.040	0.059	0.053	**0.065**	**0.074**	**0.087**	**0.093**	**0.101**
X_{APD}	0.046	0.043	0.042	0.044	0.045	0.055	0.054	0.060	**0.062**	**0.072**	**0.073**	**0.082**
Z_{EPD}	**0.073**	**0.063**	0.059	0.054	0.048	0.054	0.055	0.058	0.059	**0.067**	**0.067**	**0.072**
$LF_{\alpha,\beta}$ ...	**0.061**	0.058	0.057	0.055	0.059	**0.063**	**0.062**	**0.066**	**0.063**	**0.069**	**0.071**	**0.073**

Source: authors' work.

Table A15. $GN(\sqrt{2},b)$. PoT versus $\bar{\gamma}_2$ and $M(\sqrt{2},b;0,1)$, $n=25$

Test	$\bar{\gamma}_2=-0.5$ $b=2.779$ $M=0.947$	$\bar{\gamma}_2=-0.4$ $b=2.559$ $M=0.959$	$\bar{\gamma}_2=-0.3$ $b=2.381$ $M=0.969$	$\bar{\gamma}_2=-0.2$ $b=2.233$ $M=0.98$	$\bar{\gamma}_2=-0.1$ $b=2.108$ $M=0.99$	$\bar{\gamma}_2=0$ $b=2$ $M=1$	$\bar{\gamma}_2=0.05$ $b=1.952$ $M=0.995$	$\bar{\gamma}_2=0.1$ $b=1.907$ $M=0.99$	$\bar{\gamma}_2=0.2$ $b=1.824$ $M=0.981$	$\bar{\gamma}_2=0.3$ $b=1.752$ $M=0.972$	$\bar{\gamma}_2=0.4$ $b=1.687$ $M=0.963$	$\bar{\gamma}_2=0.5$ $b=1.628$ $M=0.954$
AD	0.050	0.047	0.043	0.044	0.048	0.046	0.048	0.054	0.054	**0.065**	**0.063**	**0.079**
SW	0.045	0.042	0.039	0.042	0.050	0.047	0.050	0.054	0.057	**0.066**	**0.072**	**0.082**
KT	0.036	0.034	0.036	0.038	0.045	0.050	0.056	0.057	**0.067**	**0.078**	**0.093**	**0.096**
AS	0.019	0.023	0.027	0.035	0.046	0.048	0.054	0.058	**0.063**	**0.075**	**0.082**	**0.089**
SF	0.027	0.030	0.035	0.039	0.049	0.050	0.056	**0.062**	**0.066**	**0.078**	**0.084**	**0.099**
AP	0.031	0.030	0.033	0.038	0.047	0.047	0.051	0.057	**0.064**	**0.075**	**0.082**	**0.090**
RJ	0.024	0.028	0.032	0.035	0.045	0.047	0.052	0.058	**0.063**	**0.074**	**0.080**	**0.094**
JB	0.014	0.019	0.026	0.033	0.045	0.048	0.054	0.059	**0.066**	**0.079**	**0.088**	**0.100**
H1	0.043	0.038	0.037	0.042	0.048	0.047	0.056	0.059	**0.061**	**0.074**	**0.080**	**0.094**
CS	0.051	0.046	0.042	0.045	0.052	0.047	0.051	0.054	0.057	**0.065**	**0.070**	**0.080**
AJB	0.013	0.016	0.028	0.031	0.040	0.051	0.056	0.057	**0.071**	**0.084**	**0.099**	**0.107**
BS	**0.062**	0.054	0.047	0.045	0.044	0.046	0.048	0.053	0.055	**0.062**	**0.069**	**0.078**
ZA	0.039	0.038	0.037	0.040	0.050	0.048	0.054	0.056	0.059	**0.069**	**0.075**	**0.084**
ZC	0.045	0.042	0.038	0.041	0.050	0.049	0.054	0.056	0.059	**0.071**	**0.072**	**0.082**
SJ	0.014	0.021	0.027	0.032	0.040	0.046	0.057	**0.061**	**0.068**	**0.084**	**0.093**	**0.112**
β_3^2	0.052	0.043	0.043	0.044	0.046	0.047	0.053	0.054	**0.062**	**0.073**	**0.077**	**0.089**
RJB	0.014	0.018	0.025	0.032	0.041	0.047	0.055	**0.060**	**0.070**	**0.084**	**0.093**	**0.107**
X_{APD}	0.045	0.041	0.040	0.043	0.046	0.048	0.053	0.057	**0.060**	**0.073**	**0.076**	**0.091**
Z_{EPD}	**0.073**	0.059	0.051	0.051	0.049	0.050	0.053	0.054	0.055	**0.067**	**0.070**	**0.078**
$LF_{\alpha,\beta}$ ...	0.056	0.056	0.051	0.051	0.055	0.046	0.049	0.053	0.050	0.059	0.054	**0.064**

Source: authors' work.

Table A16. $IH(a)$. PoT versus $\bar{\gamma}_2$ and $M(a;\mu,\sigma)$, $n=25$

Test	$\bar{\gamma}_2=-1.2$ $a=1$ $\mu=0.501$ $\sigma=0.336$ $M=0.815$	$\bar{\gamma}_2=-0.6$ $a=2$ $\mu=1$ $\sigma=0.442$ $M=0.962$	$\bar{\gamma}_2=-0.4$ $a=3$ $\mu=1.5$ $\sigma=0.525$ $M=0.983$	$\bar{\gamma}_2=-0.3$ $a=4$ $\mu=2$ $\sigma=0.598$ $M=0.989$	$\bar{\gamma}_2=-0.2$ $a=6$ $\mu=3$ $\sigma=0.723$ $M=0.992$	$\bar{\gamma}_2=-0.15$ $a=8$ $\mu=4$ $\sigma=0.83$ $M=0.994$	$\bar{\gamma}_2=-0.1$ $a=12$ $\mu=6$ $\sigma=1.011$ $M=0.996$	$\bar{\gamma}_2=-0.055$ $a=22$ $\mu=11$ $\sigma=1.362$ $M=0.998$
AD	**0.231**	0.037	0.043	0.042	0.047	0.047	0.046	0.049
SW	**0.290**	0.030	0.037	0.036	0.041	0.045	0.047	0.049
KT	**0.280**	0.027	0.025	0.030	0.036	0.039	0.039	0.045
AS	0.005	0.007	0.019	0.023	0.033	0.039	0.044	0.043
SF	**0.123**	0.015	0.025	0.029	0.038	0.044	0.047	0.046
AP	**0.244**	0.018	0.022	0.025	0.033	0.037	0.043	0.044
RJ	**0.112**	0.013	0.023	0.025	0.035	0.040	0.045	0.043
JB	0.002	0.005	0.015	0.020	0.030	0.034	0.042	0.043
H1	**0.290**	0.028	0.036	0.035	0.040	0.044	0.048	0.048
CS	**0.334**	0.035	0.039	0.040	0.043	0.045	0.049	0.050
AJB	0.001	0.005	0.013	0.021	0.032	0.037	0.036	0.046
BS	**0.288**	0.044	0.043	0.042	0.045	0.044	0.047	0.045
ZA	**0.229**	0.021	0.029	0.032	0.039	0.043	0.046	0.047
ZC	**0.333**	0.028	0.030	0.032	0.039	0.042	0.047	0.047
SJ	0.001	0.013	0.024	0.028	0.036	0.036	0.046	0.043
β_3^2	<u>**0.444**</u>	0.041	0.034	0.039	0.041	0.043	0.049	0.046
RJB	0.001	0.005	0.017	0.020	0.032	0.035	0.043	0.043
X_{APD}	**0.260**	0.028	0.033	0.034	0.041	0.042	0.044	0.047
Z_{EPD}	<u>**0.392**</u>	0.052	0.045	0.045	0.047	0.047	0.051	0.048
$LF_{\alpha,\beta}$	**0.165**	0.042	0.052	0.051	0.052	0.052	0.049	0.050

Source: authors' work.

Table A17. $NIG(a,b)$. PoT versus $\bar{\gamma}_2$ and $M(a,b;0,1)$, $n=25$

Test	$\bar{\gamma}_2=0.011$ $a=16.815$ $b=16.85$ $M=1$	$\bar{\gamma}_2=0.05$ $a=7.629$ $b=7.865$ $M=0.994$	$\bar{\gamma}_2=0.1$ $a=5.286$ $b=5.675$ $M=0.986$	$\bar{\gamma}_2=0.2$ $a=5$ $b=3$ $M=0.87$	$\bar{\gamma}_2=0.3$ $a=5$ $b=2$ $M=0.774$	$\bar{\gamma}_2=0.4$ $a=5$ $b=1.5$ $M=0.708$	$\bar{\gamma}_2=0.5$ $a=5$ $b=1.2$ $M=0.659$	$\bar{\gamma}_2=0.6$ $a=5$ $b=1$ $M=0.621$	$\bar{\gamma}_2=0.7$ $a=5$ $b=0.857$ $M=0.589$	$\bar{\gamma}_2=0.8$ $a=5$ $b=0.75$ $M=0.563$	$\bar{\gamma}_2=0.9$ $a=5$ $b=0.667$ $M=0.54$	$\bar{\gamma}_2=1$ $a=5$ $b=0.6$ $M=0.521$
AD	0.049	0.052	0.056	0.060	**0.068**	**0.069**	**0.076**	**0.076**	**0.088**	**0.088**	**0.090**	**0.108**
SW	0.049	0.055	0.060	**0.066**	**0.071**	**0.074**	**0.082**	**0.087**	**0.098**	**0.104**	**0.100**	**0.122**
KT	0.050	0.058	0.059	**0.067**	**0.077**	**0.087**	**0.093**	**0.101**	**0.113**	**0.124**	**0.138**	**0.139**
AS	0.050	0.057	**0.063**	**0.072**	**0.076**	**0.082**	**0.093**	**0.097**	**0.112**	**0.121**	**0.117**	**0.136**
SF	0.052	0.060	**0.064**	**0.075**	**0.081**	**0.085**	**0.098**	**0.104**	**0.120**	**0.129**	**0.124**	**0.152**
AP	0.049	0.057	**0.062**	**0.070**	**0.076**	**0.085**	**0.092**	**0.096**	**0.118**	**0.124**	**0.122**	**0.145**
RJ	0.048	0.057	0.059	**0.071**	**0.076**	**0.080**	**0.092**	**0.098**	**0.115**	**0.121**	**0.118**	**0.145**
JB	0.049	0.059	**0.064**	**0.075**	**0.081**	**0.089**	**0.100**	**0.105**	**0.126**	**0.133**	**0.132**	**0.154**
H1	0.050	0.056	0.057	**0.066**	**0.070**	**0.083**	**0.090**	**0.092**	**0.105**	**0.106**	**0.113**	**0.131**
CS	0.049	0.056	0.059	**0.065**	**0.070**	**0.074**	**0.078**	**0.085**	**0.096**	**0.100**	**0.098**	**0.116**
AJB	0.049	0.056	0.056	**0.067**	**0.080**	**0.092**	**0.098**	**0.110**	**0.122**	**0.136**	**0.149**	**0.149**
BS	0.045	0.053	0.053	0.057	**0.061**	**0.066**	**0.080**	**0.079**	**0.081**	**0.087**	**0.095**	**0.112**
ZA	0.049	0.057	**0.062**	**0.068**	**0.072**	**0.076**	**0.084**	**0.088**	**0.103**	**0.110**	**0.106**	**0.126**
ZC	0.048	0.055	**0.061**	**0.068**	**0.074**	**0.078**	**0.083**	**0.090**	**0.101**	**0.110**	**0.107**	**0.126**
SJ	0.048	0.055	0.058	**0.073**	**0.078**	**0.090**	**0.103**	**0.104**	**0.116**	**0.123**	**0.136**	**0.155**
β_3^2	0.050	0.056	0.056	**0.064**	**0.069**	**0.079**	**0.095**	**0.094**	**0.098**	**0.105**	**0.114**	**0.131**
RJB	0.049	0.058	**0.063**	<u>**0.077**</u>	<u>**0.083**</u>	<u>**0.091**</u>	<u>**0.106**</u>	<u>**0.111**</u>	<u>**0.131**</u>	<u>**0.135**</u>	<u>**0.141**</u>	<u>**0.164**</u>
X_{APD}	0.047	0.057	0.059	**0.067**	**0.074**	**0.082**	**0.092**	**0.095**	**0.109**	**0.115**	**0.118**	**0.137**
Z_{EPD}	0.048	0.055	0.054	**0.063**	**0.063**	**0.071**	**0.084**	**0.084**	**0.092**	**0.095**	**0.104**	**0.122**
$LF_{\alpha,\beta}$	0.053	0.049	0.053	0.057	0.059	**0.063**	**0.064**	**0.064**	**0.072**	**0.069**	**0.075**	**0.080**

Source: authors' work.

Table A18. $P(\bar{\gamma}_2)$. PoT versus $\bar{\gamma}_2$ and $M(\bar{\gamma}_2;0,\sigma)$, $n=25$

Test	$\bar{\gamma}_2=-1$ $\sigma=0.39$ $M=0.5$	$\bar{\gamma}_2=-0.9$ $\sigma=0.431$ $M=0.55$	$\bar{\gamma}_2=-0.8$ $\sigma=0.475$ $M=0.6$	$\bar{\gamma}_2=-0.7$ $\sigma=0.522$ $M=0.65$	$\bar{\gamma}_2=-0.6$ $\sigma=0.573$ $M=0.7$	$\bar{\gamma}_2=-0.5$ $\sigma=0.629$ $M=0.75$	$\bar{\gamma}_2=0$ $\sigma=1$ $M=1$	$\bar{\gamma}_2=0.1$ $\sigma=1.1$ $M=0.95$	$\bar{\gamma}_2=0.2$ $\sigma=1.213$ $M=0.9$	$\bar{\gamma}_2=0.3$ $\sigma=1.341$ $M=0.85$	$\bar{\gamma}_2=0.4$ $\sigma=1.486$ $M=0.8$	$\bar{\gamma}_2=0.5$ $\sigma=1.653$ $M=0.75$
AD	**0.098**	**0.072**	0.060	0.053	0.046	0.046	0.051	0.055	0.056	0.059	**0.061**	**0.072**
SW	**0.107**	**0.074**	0.056	0.047	0.037	0.042	0.051	0.056	0.060	**0.065**	**0.070**	**0.080**
KT	**0.126**	**0.082**	0.058	0.040	0.032	0.028	0.048	0.059	**0.070**	**0.075**	**0.086**	**0.095**
AS	0.004	0.006	0.006	0.008	0.009	0.017	0.051	0.054	**0.068**	**0.072**	**0.084**	**0.087**
SF	0.040	0.027	0.022	0.020	0.020	0.026	0.053	**0.061**	**0.067**	**0.077**	**0.083**	**0.093**
AP	**0.093**	**0.061**	0.040	0.027	0.023	0.024	0.050	0.054	**0.066**	**0.074**	**0.083**	**0.092**
RJ	0.036	0.024	0.020	0.018	0.018	0.023	0.050	0.058	**0.064**	**0.072**	**0.077**	**0.088**
JB	0.001	0.003	0.004	0.004	0.005	0.013	0.050	0.056	**0.069**	**0.076**	**0.090**	**0.097**
H1	**0.112**	**0.077**	0.056	0.043	0.037	0.037	0.051	0.056	**0.061**	**0.066**	**0.074**	**0.082**
CS	**0.128**	**0.090**	**0.066**	0.054	0.045	0.046	0.053	0.056	0.060	**0.064**	**0.068**	**0.078**
AJB	0.001	0.002	0.002	0.003	0.007	0.010	0.051	**0.063**	**0.073**	**0.080**	**0.094**	**0.099**
BS	**0.151**	**0.115**	**0.090**	**0.068**	0.060	0.050	0.047	0.050	0.056	**0.061**	**0.061**	**0.073**
ZA	**0.071**	0.048	0.036	0.032	0.028	0.034	0.052	0.059	**0.064**	**0.066**	**0.074**	**0.080**
ZC	**0.119**	**0.079**	0.056	0.043	0.036	0.039	0.051	0.055	**0.063**	**0.068**	**0.074**	**0.083**
SJ	0.002	0.004	0.005	0.006	0.010	0.015	0.047	0.055	**0.065**	**0.075**	**0.082**	**0.090**
β_3^2	**0.194**	**0.128**	**0.086**	0.060	0.049	0.043	0.047	0.056	0.060	**0.070**	**0.074**	**0.082**
RJB	0.001	0.003	0.004	0.004	0.006	0.011	0.051	0.058	**0.070**	**0.080**	**0.089**	**0.096**
X_{APD}	**0.114**	**0.079**	0.059	0.046	0.038	0.039	0.050	0.057	**0.062**	**0.070**	**0.075**	**0.085**
Z_{EPD}	**0.212**	**0.156**	**0.117**	**0.087**	**0.071**	0.060	0.049	0.054	**0.060**	**0.065**	**0.065**	**0.080**
$LF_{\alpha,\beta}$	**0.089**	**0.074**	**0.066**	0.058	0.056	0.056	0.054	0.053	0.056	0.053	0.056	**0.062**

Source: authors' work.

Table A19. $PC(a)$. PoT versus $\bar{\gamma}_2$ and $M(a;0,1)$, $n=25$

Test	$\bar{\gamma}_2=-1$ $a=1.446$ $M=0.837$	$\bar{\gamma}_2=-0.9$ $a=1.371$ $M=0.86$	$\bar{\gamma}_2=-0.8$ $a=1.307$ $M=0.881$	$\bar{\gamma}_2=-0.7$ $a=1.251$ $M=0.9$	$\bar{\gamma}_2=-0.6$ $a=1.203$ $M=0.918$	$\bar{\gamma}_2=-0.5$ $a=1.159$ $M=0.034$	$\bar{\gamma}_2=-0.4$ $a=1.121$ $M=0.949$	$\bar{\gamma}_2=-0.3$ $a=1.086$ $M=0.963$	$\bar{\gamma}_2=-0.2$ $a=1.055$ $M=0.976$	$\bar{\gamma}_2=-0.1$ $a=1.026$ $M=0.989$	$\bar{\gamma}_2=0$ $a=1$ $M=1$
AD	**0.243**	**0.171**	**0.133**	**0.100**	**0.084**	0.059	0.057	0.053	0.049	0.047	0.048
SW	**0.193**	**0.140**	**0.106**	**0.084**	**0.068**	0.050	0.048	0.046	0.043	0.048	0.046
KT	**0.185**	**0.133**	**0.099**	**0.069**	0.056	0.048	0.042	0.040	0.044	0.045	0.051
AS	0.011	0.013	0.014	0.017	0.018	0.018	0.024	0.030	0.035	0.045	0.047
SF	**0.094**	**0.064**	0.051	0.043	0.037	0.030	0.033	0.037	0.039	0.047	0.050
AP	**0.155**	**0.109**	**0.078**	0.058	0.050	0.036	0.033	0.037	0.039	0.048	0.048
RJ	**0.085**	0.059	0.047	0.039	0.034	0.027	0.030	0.034	0.035	0.044	0.047
JB	0.005	0.007	0.007	0.010	0.013	0.016	0.018	0.028	0.032	0.043	0.049
H1	**0.197**	**0.142**	**0.108**	**0.082**	**0.064**	0.047	0.047	0.045	0.045	0.047	0.050
CS	**0.223**	**0.164**	**0.124**	**0.096**	**0.079**	0.057	0.054	0.050	0.047	0.050	0.048
AJB	0.004	0.003	0.006	0.007	0.009	0.014	0.019	0.024	0.031	0.042	0.051
BS	**0.347**	**0.264**	**0.200**	**0.148**	**0.115**	**0.083**	**0.067**	**0.062**	0.049	0.046	0.052
ZA	**0.120**	**0.091**	**0.075**	**0.061**	0.050	0.040	0.040	0.041	0.041	0.049	0.047
ZC	**0.179**	**0.131**	**0.095**	**0.077**	**0.064**	0.047	0.045	0.042	0.044	0.049	0.047
SJ	0.003	0.004	0.004	0.007	0.007	0.012	0.014	0.025	0.029	0.039	0.050
β_3^2	**0.210**	**0.158**	**0.121**	**0.091**	**0.072**	0.059	0.050	0.046	0.043	0.047	0.049
RJB	0.004	0.006	0.006	0.009	0.010	0.014	0.016	0.026	0.030	0.043	0.049
X_{APD}	**0.265**	**0.186**	**0.140**	**0.100**	**0.081**	0.055	0.050	0.047	0.043	0.047	0.048
Z_{EPD}	**0.371**	**0.284**	**0.219**	**0.163**	**0.132**	**0.096**	**0.076**	**0.069**	0.057	0.051	0.051
$LF_{\alpha,\beta}$	**0.244**	**0.181**	**0.149**	**0.118**	**0.101**	**0.081**	**0.073**	**0.066**	0.058	0.059	0.058

Source: authors' work.

Table A20. $PCN(a)$. PoT versus $\bar{\gamma}_2$ and $M(a;0,1), n = 25$

Test	$\bar{\gamma}_2 =-1$ $a = 228.392$ $M = 0.5$	$\bar{\gamma}_2 =-0.9$ $a = 6.081$ $M = 0.501$	$\bar{\gamma}_2 =-0.8$ $a = 4.116$ $M = 0.52$	$\bar{\gamma}_2 =-0.7$ $a = 3.201$ $M = 0.555$	$\bar{\gamma}_2 =-0.6$ $a = 2.622$ $M = 0.595$	$\bar{\gamma}_2 =-0.5$ $a = 2.197$ $M = 0.636$	$\bar{\gamma}_2 =-0.4$ $a = 1.855$ $M = 0.677$	$\bar{\gamma}_2 =-0.3$ $a = 1.556$ $M = 0.718$	$\bar{\gamma}_2 =-0.2$ $a = 1.272$ $M = 0.762$	$\bar{\gamma}_2 =-0.1$ $a = 0.962$ $M = 0.815$	$\bar{\gamma}_2 = 0$ $a = 0$ $M = 1$
AD	**1.000**	**0.996**	**0.643**	**0.252**	**0.108**	**0.065**	0.047	0.048	0.048	0.048	0.052
SW	**1.000**	**0.990**	**0.572**	**0.217**	**0.095**	0.059	0.044	0.044	0.047	0.046	0.052
KT	**0.957**	**0.867**	**0.506**	**0.214**	**0.089**	0.052	0.041	0.038	0.040	0.042	0.056
AS	0.044	0.021	0.011	0.010	0.010	0.015	0.022	0.031	0.043	0.047	0.052
SF	**1.000**	**0.959**	**0.378**	**0.103**	0.046	0.030	0.029	0.036	0.044	0.048	0.055
AP	**1.000**	**0.890**	**0.473**	**0.176**	**0.074**	0.045	0.036	0.035	0.041	0.045	0.050
RJ	**1.000**	**0.954**	**0.357**	**0.095**	0.042	0.027	0.027	0.033	0.040	0.046	0.050
JB	**1.000**	0.012	0.004	0.005	0.006	0.012	0.019	0.027	0.038	0.045	0.051
H1	**1.000**	**0.976**	**0.567**	**0.228**	**0.099**	0.056	0.042	0.045	0.045	0.048	0.057
CS	**1.000**	**0.993**	**0.617**	**0.250**	**0.111**	**0.067**	0.048	0.047	0.049	0.047	0.053
AJB	0.025	0.004	0.002	0.004	0.006	0.009	0.018	0.024	0.037	0.042	0.054
BS	**0.985**	**0.938**	**0.706**	**0.348**	**0.165**	**0.089**	0.058	0.048	0.042	0.048	0.050
ZA	**1.000**	**0.944**	**0.405**	**0.139**	**0.070**	0.044	0.038	0.041	0.045	0.046	0.052
ZC	**1.000**	**0.981**	**0.538**	**0.205**	**0.090**	0.057	0.041	0.042	0.047	0.048	0.051
SJ	**0.111**	0.013	0.004	0.003	0.003	0.008	0.015	0.028	0.034	0.046	0.054
β_3^2	**0.985**	**0.870**	**0.518**	**0.240**	**0.120**	**0.071**	0.048	0.044	0.043	0.046	0.052
RJB	0.044	0.012	0.004	0.004	0.006	0.010	0.016	0.027	0.037	0.045	0.054
X_{APD}	**1.000**	**0.998**	**0.694**	**0.281**	**0.119**	**0.066**	0.043	0.043	0.043	0.044	0.052
Z_{EPD}	**0.957**	**0.937**	**0.726**	**0.387**	**0.194**	**0.108**	**0.071**	0.057	0.048	0.051	0.054
$LF_{\alpha,\beta}$ ...	**1.000**	**0.977**	**0.565**	**0.234**	**0.120**	**0.077**	**0.061**	0.056	0.047	0.051	0.055

Source: authors' work.

Table A21. $QG(a,b)$. PoT versus $\bar{\gamma}_2$ and $M(a,b;0,1), n = 25$

Test	$\bar{\gamma}_2 =-0.856$ $a = 0.497$ $b = 0.005$ $M = 0.779$	$\bar{\gamma}_2 =-0.8$ $a = 0.497$ $b = 0.2$ $M = 0.831$	$\bar{\gamma}_2 =-0.7$ $a = 0.497$ $b = 0.44$ $M = 0.875$	$\bar{\gamma}_2 =-0.6$ $a = 0.497$ $b = 0.6$ $M = 0.908$	$\bar{\gamma}_2 =-0.5$ $a = 0.497$ $b = 0.714$ $M = 0.932$	$\bar{\gamma}_2 =-0.4$ $a = 0.497$ $b = 0.8$ $M = 0.952$	$\bar{\gamma}_2 =-0.3$ $a = 0.497$ $b = 0.867$ $M = 0.968$	$\bar{\gamma}_2 =-0.2$ $a = 0.497$ $b = 0.92$ $M = 0.981$	$\bar{\gamma}_2 =-0.1$ $a = 0.497$ $b = 0.964$ $M = 0.992$	$\bar{\gamma}_2 =-0.02$ $a = 0.497$ $b = 0.994$ $M = 0.999$
AD	**0.066**	0.058	0.053	0.048	0.044	0.044	0.042	0.044	0.044	0.049
SW	**0.062**	0.056	0.048	0.041	0.036	0.037	0.036	0.040	0.042	0.049
KT	**0.068**	0.060	0.041	0.030	0.027	0.025	0.029	0.035	0.039	0.047
AS	0.005	0.006	0.008	0.011	0.017	0.021	0.024	0.035	0.039	0.044
SF	0.024	0.021	0.020	0.023	0.021	0.027	0.027	0.039	0.042	0.050
AP	0.048	0.041	0.033	0.024	0.023	0.026	0.028	0.035	0.039	0.046
RJ	0.022	0.020	0.018	0.022	0.019	0.024	0.026	0.036	0.039	0.047
JB	0.003	0.003	0.005	0.007	0.011	0.017	0.022	0.032	0.038	0.046
H1	**0.063**	0.057	0.044	0.040	0.035	0.036	0.035	0.041	0.042	0.048
CS	**0.077**	**0.066**	0.054	0.048	0.040	0.040	0.040	0.042	0.043	0.049
AJB	0.003	0.002	0.004	0.005	0.011	0.014	0.022	0.029	0.039	0.048
BS	**0.095**	**0.087**	**0.073**	0.055	0.049	0.046	0.046	0.041	0.041	0.045
ZA	0.042	0.038	0.033	0.031	0.029	0.032	0.031	0.040	0.043	0.047
ZC	**0.066**	0.059	0.046	0.037	0.031	0.034	0.035	0.041	0.040	0.046
SJ	0.005	0.005	0.009	0.011	0.017	0.020	0.026	0.034	0.037	0.046
β_3^2	**0.106**	**0.087**	**0.063**	0.049	0.042	0.040	0.038	0.041	0.041	0.049
RJB	0.003	0.003	0.004	0.009	0.012	0.017	0.021	0.031	0.037	0.046
X_{APD}	**0.067**	**0.060**	0.051	0.041	0.035	0.037	0.033	0.039	0.041	0.049
Z_{EPD}	**0.134**	**0.115**	**0.092**	**0.069**	0.059	0.051	0.048	0.044	0.044	0.051
$LF_{\alpha,\beta}$	**0.073**	**0.065**	**0.060**	0.057	0.049	0.050	0.048	0.048	0.049	0.054

Source: authors' work.

Table A22. $SCN(a,\omega)$. PoT versus $\bar{\gamma}_2$ and $M(a,\omega;0,1)$, $n=25$

Test	$\bar{\gamma}_2=0$ $a=1$ $\omega=1$ $M=1$	$\bar{\gamma}_2=0.05$ $a=1.139$ $\omega=0.984$ $M=0.938$	$\bar{\gamma}_2=0.1$ $a=1.203$ $\omega=0.978$ $M=0.913$	$\bar{\gamma}_2=0.2$ $a=1.302$ $\omega=0.968$ $M=0.877$	$\bar{\gamma}_2=0.3$ $a=1.387$ $\omega=0.961$ $M=0.849$	$\bar{\gamma}_2=0.4$ $a=1.466$ $\omega=0.854$ $M=0.844$	$\bar{\gamma}_2=0.5$ $a=1.543$ $\omega=0.948$ $M=0.804$	$\bar{\gamma}_2=0.6$ $a=1.618$ $\omega=0.943$ $M=0.785$	$\bar{\gamma}_2=0.7$ $a=1.694$ $\omega=0.938$ $M=0.766$	$\bar{\gamma}_2=0.8$ $a=1.771$ $\omega=0.933$ $M=0.749$	$\bar{\gamma}_2=0.9$ $a=1.85$ $\omega=0.928$ $M=0.732$	$\bar{\gamma}_2=1$ $a=1.932$ $\omega=0.923$ $M=0.716$
AD	0.050	0.047	0.047	0.051	0.052	0.054	0.056	0.056	0.053	0.053	0.057	0.057
SW	0.053	0.047	0.047	0.049	0.048	0.059	0.056	0.051	0.055	0.053	0.058	0.055
KT	0.051	0.050	0.051	0.054	0.049	0.059	0.050	0.052	0.055	0.054	**0.064**	0.056
AS	0.052	0.049	0.049	0.051	0.047	0.060	0.055	0.052	0.058	0.055	**0.060**	0.060
SF	0.053	0.049	0.050	0.052	0.051	**0.064**	0.058	0.057	0.060	0.060	**0.065**	**0.065**
AP	0.051	0.050	0.048	0.048	0.051	**0.060**	0.056	0.052	0.059	0.056	0.059	0.058
RJ	0.049	0.045	0.047	0.049	0.048	**0.060**	0.055	0.053	0.056	0.055	**0.061**	**0.061**
JB	0.051	0.049	0.050	0.048	0.050	**0.062**	0.055	0.054	**0.060**	0.059	**0.061**	**0.061**
H1	0.057	0.049	0.050	0.049	0.052	0.060	0.058	0.057	0.059	0.057	**0.064**	0.060
CS	0.053	0.048	0.047	0.049	0.048	0.059	0.057	0.052	0.055	0.053	0.059	0.055
AJB	0.053	0.049	0.050	0.052	0.052	**0.062**	0.054	0.052	0.056	0.057	**0.066**	**0.061**
BS	0.049	0.046	0.047	0.049	0.049	0.052	0.052	0.050	0.052	0.052	0.054	0.051
ZA	0.053	0.048	0.048	0.051	0.049	0.060	0.059	0.052	0.056	0.053	0.057	0.058
ZC	0.052	0.048	0.048	0.049	0.046	**0.060**	0.058	0.052	0.055	0.055	0.059	0.056
SJ	0.051	0.046	0.049	0.047	0.053	**0.063**	0.056	0.058	0.060	**0.062**	**0.066**	**0.063**
β_3^2	0.054	0.050	0.050	0.050	0.053	0.058	0.053	0.055	0.057	0.055	**0.061**	0.055
RJB	0.050	0.048	0.049	0.048	0.050	**0.062**	0.056	0.057	**0.062**	**0.062**	**0.068**	**0.062**
X_{APD} ..	0.051	0.049	0.048	0.046	0.051	0.058	0.056	0.053	0.055	0.054	**0.062**	0.057
Z_{EPD} ..	0.051	0.051	0.050	0.052	0.052	0.055	0.055	0.051	0.052	0.054	0.057	0.052
$LF_{\alpha,\beta}$	0.055	0.059	0.059	0.055	**0.064**	**0.063**	0.058	**0.062**	0.060	**0.062**	**0.064**	**0.066**

Source: authors' work.

Table A23. $SHN(a,b)$. PoT versus $\bar{\gamma}_2$ and $M(a,b;0,\sigma)$, $n=25$

Test	$\bar{\gamma}_2=-1$ $a=2.591$ $b=5.308$ $\sigma=2.344$ $M=0.55$	$\bar{\gamma}_2=-0.9$ $a=2.142$ $b=5.308$ $\sigma=2$ $M=0.555$	$\bar{\gamma}_2=-0.8$ $a=1.786$ $b=5.308$ $\sigma=1.887$ $M=0.6$	$\bar{\gamma}_2=-0.7$ $a=1.495$ $b=5.308$ $\sigma=7.217$ $M=0.653$	$\bar{\gamma}_2=-0.6$ $a=1.251$ $b=5.308$ $\sigma=5.362$ $M=0.726$	$\bar{\gamma}_2=-0.5$ $a=1.04$ $b=5.308$ $\sigma=1.591$ $M=0.75$	$\bar{\gamma}_2=-0.4$ $a=0.853$ $b=5.308$ $\sigma=1.463$ $M=0.8$	$\bar{\gamma}_2=-0.3$ $a=0.682$ $b=5.308$ $\sigma=1.309$ $M=0.851$	$\bar{\gamma}_2=-0.2$ $a=0.516$ $b=5.308$ $\sigma=1.664$ $M=0.902$	$\bar{\gamma}_2=-0.1$ $a=0.339$ $b=5.308$ $\sigma=0.958$ $M=0.967$	$\bar{\gamma}_2=-0.001$ $a=0.031$ $b=25.812$ $\sigma=0.4$ $M=1$
AD	**0.133**	**0.092**	**0.073**	**0.061**	0.054	0.041	0.045	0.045	0.045	0.046	0.048
SW	**0.130**	**0.086**	**0.067**	0.056	0.045	0.036	0.038	0.038	0.040	0.047	0.045
KT	**0.138**	**0.090**	**0.067**	0.046	0.037	0.033	0.031	0.031	0.038	0.043	0.051
AS	0.006	0.008	0.009	0.012	0.013	0.014	0.019	0.027	0.033	0.043	0.047
SF	0.051	0.036	0.030	0.024	0.024	0.021	0.028	0.033	0.037	0.045	0.050
AP	**0.113**	**0.075**	0.050	0.037	0.030	0.025	0.025	0.031	0.034	0.045	0.048
RJ	0.046	0.032	0.027	0.021	0.022	0.020	0.025	0.030	0.034	0.043	0.047
JB	0.003	0.003	0.004	0.007	0.009	0.012	0.015	0.024	0.030	0.043	0.049
H1	**0.142**	**0.096**	**0.069**	0.052	0.042	0.033	0.036	0.040	0.042	0.046	0.050
CS	**0.155**	**0.105**	**0.078**	**0.064**	0.051	0.040	0.041	0.041	0.042	0.048	0.047
AJB	0.002	0.002	0.003	0.005	0.005	0.011	0.016	0.020	0.029	0.041	0.051
BS	**0.201**	**0.140**	**0.106**	**0.080**	**0.069**	0.051	0.045	0.049	0.045	0.044	0.052
ZA	**0.085**	0.059	0.048	0.038	0.034	0.028	0.032	0.035	0.038	0.047	0.047
ZC	**0.135**	**0.091**	**0.066**	0.054	0.042	0.032	0.034	0.036	0.040	0.046	0.047
SJ	0.001	0.003	0.005	0.007	0.011	0.016	0.019	0.029	0.034	0.041	0.050
β_3^2	**0.197**	**0.139**	**0.100**	**0.072**	0.054	0.046	0.040	0.043	0.041	0.046	0.049
RJB	0.002	0.003	0.004	0.007	0.008	0.013	0.016	0.026	0.030	0.043	0.049
X_{APD} ...	**0.155**	**0.101**	**0.073**	0.057	0.044	0.034	0.036	0.038	0.039	0.044	0.048
Z_{EPD} ...	**0.258**	**0.185**	**0.136**	**0.104**	**0.085**	**0.062**	0.052	0.054	0.048	0.048	0.051
$LF_{\alpha,\beta}$	**0.127**	**0.092**	**0.075**	**0.069**	**0.063**	0.054	0.057	0.059	0.053	0.058	0.057

Source: authors' work.

Table A24. $SB(a,4)$. PoT versus $\bar{\gamma}_2$ and $M(v;0,\sigma)$, $n=25$

Test	$\bar{\gamma}_2=-1$ $a=0.84$ $\sigma=0.738$ $M=0.5$	$\bar{\gamma}_2=-0.9$ $a=0.952$ $\sigma=0.747$ $M=0.55$	$\bar{\gamma}_2=-0.8$ $a=1.079$ $\sigma=0.751$ $M=0.6$	$\bar{\gamma}_2=-0.7$ $a=1.225$ $\sigma=0.749$ $M=0.65$	$\bar{\gamma}_2=-0.6$ $a=1.398$ $\sigma=0.739$ $M=0.7$	$\bar{\gamma}_2=-0.5$ $a=1.613$ $\sigma=0.718$ $M=0.75$	$\bar{\gamma}_2=-0.4$ $a=1.892$ $\sigma=0.685$ $M=0.8$	$\bar{\gamma}_2=-0.3$ $a=2.286$ $\sigma=0.633$ $M=0.85$	$\bar{\gamma}_2=-0.2$ $a=2.923$ $\sigma=0.552$ $M=0.9$	$\bar{\gamma}_2=-0.1$ $a=4.305$ $\sigma=0.417$ $M=0.95$	$\bar{\gamma}_2=-0.01$ $a=19.645$ $\sigma=0.102$ $M=1$
AD	**0.105**	**0.074**	**0.061**	0.054	0.049	0.039	0.044	0.045	0.045	0.046	0.048
SW	**0.110**	**0.074**	0.057	0.049	0.040	0.034	0.036	0.038	0.040	0.046	0.045
KT	**0.119**	**0.076**	0.058	0.040	0.032	0.031	0.029	0.031	0.038	0.043	0.050
AS	0.005	0.006	0.007	0.010	0.012	0.013	0.018	0.027	0.033	0.043	0.047
SF	0.041	0.028	0.025	0.021	0.022	0.020	0.027	0.032	0.037	0.045	0.050
AP	**0.098**	**0.062**	0.044	0.030	0.026	0.022	0.024	0.030	0.034	0.044	0.047
RJ	0.038	0.026	0.022	0.019	0.019	0.019	0.025	0.030	0.034	0.043	0.047
JB	0.002	0.002	0.004	0.006	0.007	0.011	0.015	0.024	0.030	0.043	0.048
H1	**0.118**	**0.078**	0.058	0.045	0.038	0.030	0.035	0.039	0.042	0.046	0.050
CS	**0.134**	**0.090**	**0.069**	0.057	0.047	0.039	0.040	0.040	0.041	0.048	0.047
AJB	0.001	0.001	0.003	0.005	0.004	0.010	0.015	0.020	0.029	0.041	0.051
BS	**0.162**	**0.116**	**0.090**	**0.072**	**0.063**	0.048	0.044	0.049	0.045	0.044	0.051
ZA	**0.074**	0.050	0.041	0.033	0.031	0.026	0.031	0.035	0.038	0.047	0.047
ZC	**0.120**	**0.077**	0.059	0.047	0.038	0.030	0.033	0.035	0.040	0.046	0.047
SJ	0.001	0.004	0.005	0.008	0.011	0.017	0.019	0.030	0.034	0.041	0.050
β_3^2	**0.191**	**0.126**	**0.089**	**0.066**	0.049	0.042	0.038	0.042	0.040	0.046	0.048
RJB	0.002	0.002	0.004	0.006	0.007	0.013	0.016	0.026	0.030	0.043	0.048
X_{APD}	**0.122**	**0.081**	**0.061**	0.047	0.038	0.031	0.035	0.037	0.038	0.044	0.048
Z_{EPD}	**<u>0.221</u>**	**<u>0.156</u>**	**<u>0.116</u>**	**<u>0.091</u>**	**<u>0.075</u>**	0.057	0.050	0.052	0.048	0.048	0.051
$LF_{\alpha,\beta}$	**0.097**	**0.074**	**0.065**	**0.063**	**0.061**	0.052	0.056	0.058	0.053	0.058	0.057

Source: authors' work.

Table A25. $SU(a,17)$. PoT versus $\bar{\gamma}_2$ and $M(v;0,\sigma)$, $n=25$

Test	$\bar{\gamma}_2=0.01$ $a=20.033$ $\sigma=0.849$ $M=1$	$\bar{\gamma}_2=0.05$ $a=9.027$ $\sigma=1.792$ $M=0.975$	$\bar{\gamma}_2=0.1$ $a=6.441$ $\sigma=2.389$ $M=0.95$	$\bar{\gamma}_2=0.2$ $a=4.634$ $\sigma=3.001$ $M=0.9$	$\bar{\gamma}_2=0.3$ $a=3.846$ $\sigma=3.261$ $M=0.85$	$\bar{\gamma}_2=0.4$ $a=3.384$ $\sigma=3.335$ $M=0.8$	$\bar{\gamma}_2=0.5$ $a=3.072$ $\sigma=3.295$ $M=0.75$	$\bar{\gamma}_2=0.6$ $a=2.845$ $\sigma=3.181$ $M=0.7$	$\bar{\gamma}_2=0.7$ $a=2.67$ $\sigma=3.016$ $M=0.65$	$\bar{\gamma}_2=0.8$ $a=2.531$ $\sigma=2.816$ $M=0.6$	$\bar{\gamma}_2=0.9$ $a=2.417$ $\sigma=2.594$ $M=0.55$	$\bar{\gamma}_2=1$ $a=2.321$ $\sigma=2.357$ $M=0.5$
AD	0.048	0.049	0.055	0.059	**0.063**	**0.062**	**0.072**	**0.080**	**0.081**	**0.089**	**0.093**	**0.096**
SW	0.048	0.053	0.059	**0.064**	**0.070**	**0.071**	**0.083**	**0.085**	**0.091**	**0.104**	**0.109**	**0.108**
KT	0.053	0.054	**0.063**	**0.066**	**0.080**	**0.086**	**0.092**	**0.103**	**0.112**	**0.123**	**0.131**	**0.139**
AS	0.051	0.054	**0.064**	**0.069**	**0.074**	**0.082**	**0.094**	**0.099**	**0.106**	**0.113**	**0.121**	**0.125**
SF	0.051	0.059	**0.067**	**0.072**	**0.076**	**0.087**	**0.098**	**0.102**	**0.111**	**0.122**	**0.129**	**0.135**
AP	0.050	0.057	**0.064**	**0.070**	**0.073**	**0.086**	**0.096**	**0.104**	**0.109**	**0.116**	**0.126**	**0.132**
RJ	0.048	0.055	**0.063**	**0.068**	**0.073**	**0.081**	**0.093**	**0.097**	**0.106**	**0.116**	**0.124**	**0.128**
JB	0.051	0.056	**0.065**	**0.072**	**0.075**	**0.093**	**0.101**	**0.109**	**0.116**	**0.124**	**0.135**	**0.142**
H1	0.049	0.053	**0.061**	**0.066**	**0.069**	**0.074**	**0.082**	**0.092**	**0.098**	**0.104**	**0.114**	**0.117**
CS	0.049	0.054	0.057	**0.063**	**0.068**	**0.070**	**0.082**	**0.083**	**0.090**	**0.101**	**0.106**	**0.104**
AJB	0.054	0.052	**<u>0.064</u>**	**<u>0.072</u>**	**<u>0.084</u>**	**<u>0.092</u>**	**<u>0.101</u>**	**<u>0.109</u>**	**<u>0.121</u>**	**<u>0.131</u>**	**<u>0.140</u>**	**<u>0.150</u>**
BS	0.046	0.046	0.052	0.059	0.059	**0.066**	**0.068**	**0.079**	**0.080**	**0.089**	**0.095**	**0.098**
ZA	0.049	0.056	**0.061**	**0.065**	**0.069**	**0.074**	**0.088**	**0.089**	**0.097**	**0.107**	**0.114**	**0.114**
ZC	0.049	0.054	**0.061**	**0.065**	**0.070**	**0.076**	**0.088**	**0.088**	**0.098**	**0.106**	**0.115**	**0.114**
SJ	0.051	0.055	**0.063**	**0.071**	**0.073**	**0.088**	**0.092**	**0.104**	**0.108**	**0.120**	**0.128**	**0.140**
β_3^2	0.049	0.053	0.057	**0.064**	**0.067**	**0.078**	**0.080**	**0.092**	**0.092**	**0.105**	**0.111**	**0.115**
RJB	0.049	0.057	**0.066**	**0.072**	**0.073**	**<u>0.093</u>**	**<u>0.101</u>**	**<u>0.113</u>**	**<u>0.119</u>**	**<u>0.129</u>**	**<u>0.140</u>**	**<u>0.148</u>**
X_{APD}	0.047	0.054	**0.061**	**0.066**	**0.068**	**0.077**	**0.086**	**0.095**	**0.103**	**0.112**	**0.117**	**0.124**
Z_{EPD}	0.048	0.052	0.056	0.060	**0.064**	**0.070**	**0.074**	**0.084**	**0.088**	**0.099**	**0.104**	**0.109**
$LF_{\alpha,\beta}$ ...	**0.060**	0.060	**0.064**	**0.064**	**0.070**	**0.066**	**0.074**	**0.085**	**0.078**	**0.087**	**0.089**	**0.089**

Source: authors' work.

Table A26. $T(v)$. PoT versus $\bar{\gamma}_2$ and $M(v;0,1)$, $n=25$

Test	$\bar{\gamma}_2=0.01$ $v=604$ $M=0.999$	$\bar{\gamma}_2=0.05$ $v=124$ $M=0.997$	$\bar{\gamma}_2=0.1$ $v=64$ $M=0.995$	$\bar{\gamma}_2=0.2$ $v=34$ $M=0.991$	$\bar{\gamma}_2=0.3$ $v=24$ $M=0.987$	$\bar{\gamma}_2=0.4$ $v=19$ $M=0.983$	$\bar{\gamma}_2=0.5$ $v=16$ $M=0.98$	$\bar{\gamma}_2=0.6$ $v=14$ $M=0.978$	$\bar{\gamma}_2=0.7$ $v=12.571$ $M=0.975$	$\bar{\gamma}_2=0.8$ $v=11.5$ $M=0.973$	$\bar{\gamma}_2=0.9$ $v=10.667$ $M=0.971$	$\bar{\gamma}_2=1$ $v=10$ $M=0.969$
AD	0.051	0.054	0.053	0.056	**0.065**	**0.066**	**0.067**	**0.075**	**0.081**	**0.088**	**0.090**	**0.091**
SW	0.048	0.054	0.059	**0.062**	**0.071**	**0.074**	**0.076**	**0.085**	**0.093**	**0.100**	**0.104**	**0.105**
KT	0.053	0.056	**0.062**	**0.065**	**0.078**	**0.088**	**0.094**	**0.103**	**0.110**	**0.111**	**0.124**	**0.134**
AS	0.050	0.057	**0.060**	**0.067**	**0.078**	**0.084**	**0.087**	**0.096**	**0.105**	**0.113**	**0.121**	**0.120**
SF	0.054	0.056	**0.063**	**0.069**	**0.079**	**0.086**	**0.090**	**0.100**	**0.111**	**0.120**	**0.126**	**0.126**
AP	0.049	0.058	**0.061**	**0.064**	**0.079**	**0.086**	**0.090**	**0.099**	**0.111**	**0.115**	**0.125**	**0.122**
RJ	0.049	0.053	0.059	**0.066**	**0.075**	**0.082**	**0.085**	**0.095**	**0.106**	**0.114**	**0.120**	**0.121**
JB	0.051	0.057	**0.063**	**0.068**	**0.081**	**0.088**	**0.094**	**0.105**	**0.118**	**0.122**	**0.133**	**0.131**
H1	0.052	0.055	0.058	**0.063**	**0.073**	**0.077**	**0.083**	**0.086**	**0.096**	**0.105**	**0.109**	**0.109**
CS	0.049	0.056	0.059	**0.061**	**0.071**	**0.072**	**0.076**	**0.081**	**0.091**	**0.098**	**0.101**	**0.103**
AJB	0.055	0.057	**0.064**	**0.069**	**0.086**	**0.091**	**0.098**	**0.110**	**0.119**	**0.122**	**0.133**	**0.141**
BS	0.048	0.047	0.055	0.050	**0.060**	**0.065**	**0.070**	**0.073**	**0.084**	**0.089**	**0.094**	**0.094**
ZA	0.052	0.054	**0.064**	**0.066**	**0.072**	**0.077**	**0.082**	**0.087**	**0.099**	**0.106**	**0.108**	**0.111**
ZC	0.050	0.056	**0.064**	**0.064**	**0.074**	**0.079**	**0.080**	**0.088**	**0.098**	**0.106**	**0.111**	**0.111**
SJ	0.050	0.051	0.058	**0.065**	**0.075**	**0.086**	**0.091**	**0.099**	**0.111**	**0.118**	**0.128**	**0.124**
β_3^2	0.054	0.054	0.058	**0.060**	**0.067**	**0.074**	**0.084**	**0.085**	**0.095**	**0.102**	**0.108**	**0.112**
RJB	0.051	0.055	**0.064**	**0.069**	**0.082**	**0.092**	**0.096**	**0.108**	**0.122**	**0.125**	**0.136**	**0.137**
X_{APD}	0.051	0.053	0.059	**0.064**	**0.074**	**0.079**	**0.084**	**0.091**	**0.102**	**0.113**	**0.116**	**0.118**
Z_{EPD}	0.053	0.053	0.059	0.057	**0.066**	**0.071**	**0.074**	**0.081**	**0.091**	**0.097**	**0.100**	**0.105**
$LF_{\alpha,\beta}$	0.059	**0.062**	**0.063**	**0.062**	**0.066**	**0.074**	**0.071**	**0.076**	**0.078**	**0.087**	**0.087**	**0.087**

Source: authors' work.

Table A27. $TPSN(1,b)$. PoT versus $\bar{\gamma}_2$ and $M(1,b;0,1)$, $n=25$

Test	$\bar{\gamma}_2=-0.388$ $b=1.214$ $M=0.916$	$\bar{\gamma}_2=-0.35$ $b=0.765$ $M=0.922$	$\bar{\gamma}_2=-0.3$ $b=0.574$ $M=0.93$	$\bar{\gamma}_2=-0.25$ $b=0.443$ $M=0.939$	$\bar{\gamma}_2=-0.2$ $b=0.337$ $M=0.949$	$\bar{\gamma}_2=-0.15$ $b=0.245$ $M=0.961$	$\bar{\gamma}_2=-0.1$ $b=0.16$ $M=0.972$	$\bar{\gamma}_2=-0.05$ $b=0.079$ $M=0.986$	$\bar{\gamma}_2=0$ $b=0$ $M=1$
AD	**0.063**	0.056	0.049	0.049	0.048	0.049	0.046	0.046	0.049
SW	0.056	0.049	0.049	0.047	0.048	0.046	0.046	0.043	0.048
KT	0.042	0.045	0.040	0.039	0.040	0.041	0.044	0.048	0.046
AS	0.029	0.028	0.032	0.032	0.035	0.039	0.043	0.039	0.049
SF	0.039	0.036	0.036	0.038	0.040	0.041	0.044	0.044	0.051
AP	0.041	0.036	0.038	0.038	0.042	0.040	0.045	0.043	0.052
RJ	0.036	0.034	0.033	0.035	0.037	0.038	0.042	0.041	0.047
JB	0.021	0.022	0.027	0.027	0.034	0.036	0.043	0.042	0.051
H1	0.052	0.046	0.042	0.047	0.043	0.048	0.045	0.044	0.051
CS	**0.061**	0.056	0.053	0.051	0.051	0.049	0.049	0.044	0.050
AJB	0.019	0.025	0.025	0.024	0.029	0.034	0.038	0.048	0.047
BS	**0.076**	**0.066**	0.055	0.056	0.053	0.051	0.046	0.047	0.048
ZA	0.048	0.045	0.042	0.044	0.045	0.045	0.047	0.042	0.050
ZC	0.051	0.047	0.048	0.046	0.046	0.046	0.048	0.043	0.050
SJ	0.014	0.016	0.020	0.023	0.026	0.033	0.038	0.045	0.052
β_3^2	0.055	0.048	0.048	0.049	0.043	0.047	0.047	0.046	0.053
RJB	0.018	0.019	0.023	0.025	0.029	0.034	0.040	0.043	0.050
X_{APD}	0.058	0.050	0.044	0.045	0.047	0.047	0.045	0.043	0.051
Z_{EPD}	**0.085**	**0.074**	**0.065**	**0.065**	0.060	0.056	0.052	0.048	0.053
$LF_{\alpha,\beta}$	**0.076**	**0.071**	**0.064**	**0.061**	0.058	0.058	0.056	0.055	0.058

Source: authors' work.

Table A28. $TN(a, b)$. PoT versus $\bar{\gamma}_2$ and $M(a, b; 0, 1)$, $n = 25$

Test	$\bar{\gamma}_2 = -1$ $a = 0.674$ $b = 0.568$ $M = 0.5$	$\bar{\gamma}_2 = -0.9$ $a = 0.755$ $b = 0.522$ $M = 0.55$	$\bar{\gamma}_2 = -0.8$ $a = 0.856$ $b = 0.512$ $M = 0.6$	$\bar{\gamma}_2 = -0.7$ $a = 0.995$ $b = 0.531$ $M = 0.65$	$\bar{\gamma}_2 = -0.6$ $a = 1.172$ $b = 0.567$ $M = 0.7$	$\bar{\gamma}_2 = -0.5$ $a = 1.388$ $b = 0.616$ $M = 0.75$	$\bar{\gamma}_2 = -0.4$ $a = 1.651$ $b = 0.674$ $M = 0.8$	$\bar{\gamma}_2 = -0.3$ $a = 1.973$ $b = 0.741$ $M = 0.85$	$\bar{\gamma}_2 = -0.2$ $a = 2.381$ $b = 0.817$ $M = 0.9$	$\bar{\gamma}_2 = -0.1$ $a = 2.948$ $b = 0.903$ $M = 0.95$	$\bar{\gamma}_2 = 0$ $a = 5$ $b = 1$ $M = 1$
AD	**0.094**	**0.064**	0.052	0.045	0.040	0.043	0.042	0.040	0.042	0.048	0.048
SW	**0.105**	**0.066**	0.050	0.041	0.032	0.036	0.034	0.036	0.039	0.046	0.047
KT	**0.122**	**0.075**	0.050	0.033	0.024	0.021	0.021	0.025	0.029	0.035	0.053
AS	0.004	0.004	0.005	0.007	0.008	0.014	0.019	0.023	0.032	0.042	0.048
SF	0.038	0.021	0.018	0.019	0.017	0.021	0.026	0.028	0.035	0.044	0.052
AP	**0.086**	0.055	0.038	0.021	0.018	0.019	0.020	0.024	0.029	0.041	0.050
RJ	0.033	0.018	0.017	0.018	0.016	0.019	0.023	0.025	0.033	0.041	0.048
JB	0.001	0.002	0.002	0.004	0.004	0.010	0.014	0.019	0.026	0.039	0.051
H1	**0.105**	**0.065**	0.046	0.036	0.029	0.033	0.033	0.032	0.041	0.046	0.051
CS	**0.126**	**0.081**	0.059	0.047	0.035	0.039	0.037	0.037	0.041	0.047	0.048
AJB	0.001	0.001	0.002	0.003	0.006	0.007	0.012	0.018	0.027	0.035	0.053
BS	**0.141**	**0.102**	**0.078**	0.055	0.048	0.043	0.038	0.039	0.039	0.045	0.048
ZA	**0.070**	0.043	0.032	0.029	0.025	0.028	0.028	0.031	0.036	0.045	0.048
ZC	**0.118**	**0.072**	0.050	0.040	0.027	0.032	0.030	0.031	0.036	0.045	0.049
SJ	0.002	0.003	0.005	0.007	0.010	0.020	0.024	0.026	0.033	0.045	0.050
β_3^2	**0.197**	**0.126**	**0.084**	0.054	0.038	0.039	0.033	0.034	0.038	0.048	0.051
RJB	0.001	0.003	0.003	0.004	0.005	0.012	0.016	0.022	0.029	0.040	0.052
X_{APD}	**0.104**	**0.066**	0.049	0.037	0.029	0.032	0.030	0.031	0.036	0.045	0.049
Z_{EPD}	**0.199**	**0.142**	**0.104**	**0.074**	0.058	0.049	0.042	0.043	0.040	0.049	0.050
$LF_{\alpha,\beta}$	**0.087**	**0.064**	0.060	0.055	0.049	0.053	0.054	0.050	0.053	0.059	0.056

Source: authors' work.

Table A29. $TU(\lambda)$. PoT versus $\bar{\gamma}_2$, $n = 25$

Test	$\bar{\gamma}_2 = -0.8$ $\lambda = 0.409$	$\bar{\gamma}_2 = -0.7$ $\lambda = 0.351$	$\bar{\gamma}_2 = -0.6$ $\lambda = 0.303$	$\bar{\gamma}_2 = -0.5$ $\lambda = 0.264$	$\bar{\gamma}_2 = -0.4$ $\lambda = 0.23$	$\bar{\gamma}_2 = 0$ $\lambda = 0.135$	$\bar{\gamma}_2 = 0.05$ $\lambda = 0.126$	$\bar{\gamma}_2 = 0.1$ $\lambda = 0.117$	$\bar{\gamma}_2 = 0.2$ $\lambda = 0.102$	$\bar{\gamma}_2 = 0.3$ $\lambda = 0.087$	$\bar{\gamma}_2 = 0.4$ $\lambda = 0.074$	$\bar{\gamma}_2 = 0.5$ $\lambda = 0.062$
AD	**0.060**	0.053	0.046	0.041	0.046	0.048	0.051	0.060	0.059	**0.061**	**0.063**	**0.073**
SW	0.056	0.048	0.036	0.036	0.039	0.051	0.049	0.058	**0.062**	**0.066**	**0.073**	**0.080**
KT	0.055	0.042	0.034	0.030	0.027	0.047	0.057	0.055	**0.065**	**0.074**	**0.085**	**0.097**
AS	0.007	0.009	0.010	0.015	0.021	0.050	0.053	0.059	**0.065**	**0.073**	**0.085**	**0.092**
SF	0.022	0.022	0.019	0.023	0.029	0.053	0.056	**0.062**	**0.072**	**0.075**	**0.088**	**0.098**
AP	0.039	0.032	0.025	0.021	0.026	0.049	0.053	0.059	**0.066**	**0.074**	**0.088**	**0.095**
RJ	0.020	0.021	0.018	0.020	0.027	0.049	0.052	0.059	**0.067**	**0.072**	**0.083**	**0.094**
JB	0.004	0.005	0.007	0.010	0.016	0.050	0.053	**0.063**	**0.067**	**0.078**	**0.093**	**0.102**
H1	0.055	0.045	0.036	0.034	0.037	0.055	0.054	0.058	**0.065**	**0.068**	**0.077**	**0.087**
CS	**0.066**	0.057	0.043	0.040	0.044	0.052	0.049	0.057	0.060	**0.065**	**0.071**	**0.079**
AJB	0.002	0.004	0.006	0.010	0.013	0.049	0.057	0.057	**0.071**	**0.084**	**0.090**	**0.105**
BS	**0.087**	**0.071**	0.059	0.051	0.045	0.048	0.049	0.050	0.057	0.056	**0.065**	**0.075**
ZA	0.038	0.033	0.029	0.030	0.035	0.051	0.050	0.056	**0.061**	**0.071**	**0.078**	**0.085**
ZC	0.058	0.047	0.035	0.034	0.037	0.050	0.051	0.058	**0.062**	**0.068**	**0.079**	**0.085**
SJ	0.005	0.007	0.011	0.015	0.021	0.050	0.056	**0.063**	**0.073**	**0.074**	**0.087**	**0.100**
β_3^2	**0.087**	**0.066**	0.052	0.044	0.041	0.051	0.054	0.059	**0.064**	**0.067**	**0.078**	**0.087**
RJB	0.003	0.005	0.007	0.011	0.017	0.051	0.056	**0.066**	**0.071**	**0.080**	**0.092**	**0.104**
X_{APD}	0.059	0.047	0.039	0.036	0.036	0.050	0.053	0.057	**0.064**	**0.068**	**0.077**	**0.090**
Z_{EPD}	**0.113**	**0.091**	**0.071**	0.058	0.053	0.050	0.051	0.052	**0.061**	0.059	**0.068**	**0.076**
$LF_{\alpha,\beta}$	**0.069**	0.060	0.054	0.050	0.053	0.051	0.052	0.054	0.057	0.054	0.055	0.060

Source: authors' work.

Table A30. $UP(4/3, b)$. PoT versus $\bar{\gamma}_2$ and $M(\frac{4}{3},\ b; 0, 1), n = 25$

Test	$\bar{\gamma}_2 = -2$ $b = 44$ $M = 0.032$	$\bar{\gamma}_2 = -1.996$ $b = 15$ $M = 0.077$	$\bar{\gamma}_2 = -1.992$ $b = 10$ $M = 0.106$	$\bar{\gamma}_2 = -1.989$ $b = 8$ $M = 0.126$	$\bar{\gamma}_2 = -1.982$ $b = 6$ $M = 0.155$	$\bar{\gamma}_2 = -1.976$ $b = 5$ $M = 0.177$	$\bar{\gamma}_2 = -1.966$ $b = 4$ $M = 0.207$	$\bar{\gamma}_2 = -1.948$ $b = 3$ $M = 0.249$	$\bar{\gamma}_2 = -1.911$ $b = 2$ $M = 0.318$	$\bar{\gamma}_2 = -1.81$ $b = 1$ $M = 0.446$
AD	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
SW	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
KT	**0.958**	**0.962**	**0.955**	**0.955**	**0.959**	**0.959**	**0.962**	**0.956**	**0.948**	**0.925**
AS	0.045	0.044	0.043	0.042	0.042	0.046	0.044	0.045	0.042	0.029
SF	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
AP	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**0.998**	**0.991**	**0.976**	**0.961**
RJ	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
JB	**1.000**	**0.645**	**0.406**	**0.311**	**0.243**	**0.216**	**0.176**	**0.127**	**0.080**	0.030
H1	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
CS	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
AJB	0.025	0.023	0.025	0.022	0.019	0.017	0.016	0.014	0.014	0.006
BS	**0.984**	**0.986**	**0.986**	**0.986**	**0.985**	**0.984**	**0.986**	**0.984**	**0.977**	**0.961**
ZA	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
ZC	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
SJ	**0.110**	**0.109**	**0.105**	**0.101**	**0.091**	**0.082**	**0.066**	0.052	0.037	0.016
β_3^2	**0.984**	**0.986**	**0.985**	**0.986**	**0.984**	**0.983**	**0.984**	**0.980**	**0.981**	**0.976**
RJB	0.045	0.044	0.043	0.042	0.042	0.046	0.043	0.043	0.029	0.014
X_{APD}	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
Z_{EPD}	**0.955**	**0.957**	**0.958**	**0.959**	**0.959**	**0.955**	**0.958**	**0.955**	**0.959**	**0.959**
$LF_{\alpha,\beta}$	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**0.998**

Source: authors' work.